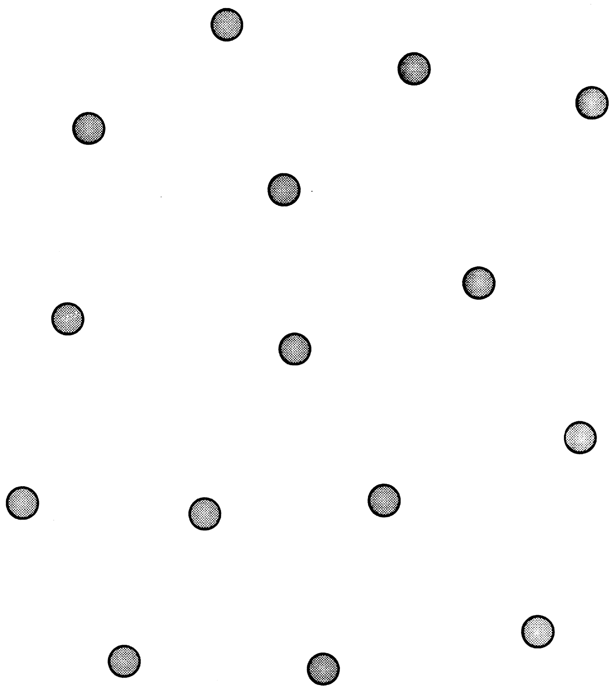

Fig. 2. Highly porous medium consisting of disperse spheres with inter-spherical spacing greater than five sphere diameters.

and even Eq. (59). The latter equation is a subject in the study by Travkin and Kushch [59] dealing with the Stokes flow and heat transport in regular and irregular conglomerates of spherical particles, Fig. 3, where a rigorous solution applied to porous-media problems in VAT statements.

Few equations of heat transfer with constant conductivities in porous media will depict the benefits of the specific morphologies that are now being used to deal with these problems. To develop a one-dimensional steady-state fluid phase equation of convective heat transfer in porous media, one needs to start from the equation in deviation form

$$\langle m\rangle \tilde{U}_i \nabla \tilde{T}_f = \nabla\langle -\hat{T}_f \hat{u}_i\rangle_f + a_f \nabla\cdot\left[\frac{1}{\Delta\Omega}\int_{\partial S_w}\hat{T}_f \vec{ds}\right] + \frac{a_f}{\Delta\Omega}\int_{\partial S_w}\nabla\hat{T}_f\cdot\vec{ds} + \frac{\langle m\rangle}{(\varrho c_p)_f}S_{T_f}, \tag{66}$$

or the full value equation

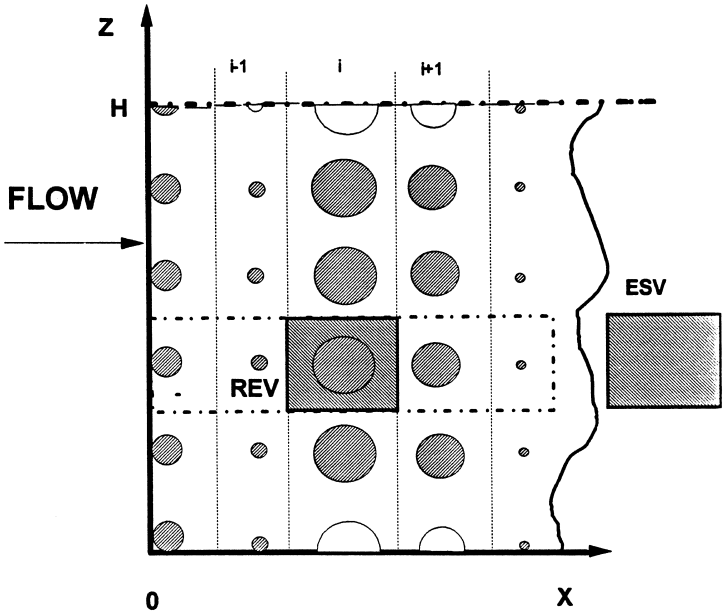

Fig. 3. Randomized series of spherical bead screens — one-dimensional globular morphology.

$$\langle m\rangle \tilde{U}_i \nabla \tilde{T}_f = \nabla\langle -\hat{T}_f \hat{u}_i\rangle_f + a_f \nabla\nabla\left(\langle m\rangle \tilde{T}_f\right) + a_f \nabla\cdot\left[\frac{1}{\Delta\Omega}\int_{\partial S_w} T_f \vec{ds}\right] + \frac{a_f}{\Delta\Omega}\nabla T_f\cdot\vec{ds} + \frac{\langle m\rangle}{(\varrho c_p)_f} S_{T_f}. \quad (67)$$

It is interesting to note that Hsu and Cheng [32] dropped the morpho-convective term

$$\varrho_f \nabla\langle -\hat{u}_i \hat{u}_i\rangle_f$$

in the averaged Navier–Stokes momentum equation and at the same time acquired the analogous term

$$c_{pf\varrho f}\nabla\langle -\hat{T}_f \hat{u}_i\rangle_f$$

in the averaged heat-transfer equation. To close this term, Hsu and Cheng [32] used several assumptions to comply with the closure schemes developed by Zanotti and Carbonell [1] and Carbonell and Whitaker [19]. Numerical results were obtained using experimental measurements of the bulk stagnant conductivity, k_d, and the tensorial quantity of the porous medium bulk thermal dispersivity, k.

When the coefficient of thermal conductivity, k_f, is a constant value, the steady state conduction regime is described by

$$k_f \nabla^2\left(\langle m\rangle \tilde{T}_f\right) + \nabla\cdot\left[\frac{k_f}{\Delta\Omega}\int_{\partial S_w} T_f \vec{ds}\right] + \frac{k_f}{\Delta\Omega}\int_{\partial S_w} \nabla T_f \cdot \vec{ds} + \langle m\rangle S_{T_f} = 0, \tag{68}$$

or written in fluctuation variables it is

$$k_f \nabla\cdot\left[\frac{1}{\Delta\Omega}\int_{\partial S_w} \hat{T}_f \vec{ds}\right] + \frac{k_f}{\Delta\Omega}\int_{\partial S_w} \nabla \hat{T}_f \cdot \vec{ds} + \langle m\rangle S_{T_f} = 0. \tag{69}$$

The one-dimensional version of the equations, without a source in a motionless matrix in Cartesian coordinates, are

$$\frac{\partial}{\partial x}\left[\langle m\rangle \frac{\partial \tilde{T}_f}{\partial x}\right] + \frac{\partial}{\partial x}\left[\frac{1}{\Delta\Omega}\int_{\partial S_w} \hat{T}_f \vec{ds}\right] + \frac{1}{\Delta\Omega}\int_{\partial S_w} \frac{\partial T_f}{\partial x_i}\cdot \vec{ds} = 0, \tag{70}$$

or

$$\frac{\partial}{\partial x}\left[\frac{1}{\Delta\Omega}\int_{\partial S_w} \hat{T}_f \vec{ds}\right] + \frac{1}{\Delta\Omega}\int_{\partial S_w} \frac{\partial \hat{T}_f}{\partial x_i}\cdot \vec{ds} = 0, \tag{71}$$

or

$$\frac{\partial^2}{\partial x^2}\left[\langle m\rangle \tilde{T}_f\right] + \frac{\partial}{\partial x}\left[\frac{1}{\Delta\Omega}\int_{\partial S_w} T_f \vec{ds}\right] + \frac{1}{\Delta\Omega}\int_{\partial S_w} \frac{\partial T_f}{\partial x_i}\cdot \vec{ds} = 0. \tag{72}$$

Meanwhile, in the solid phase with constant k_s, the equation yields the same form

$$\frac{\partial}{\partial x}\left[\langle s\rangle \frac{\partial \{T_s\}_s}{\partial x}\right] + \frac{\partial}{\partial x}\left[\frac{1}{\Delta\Omega}\int_{\partial S_w} \hat{T}_s \vec{ds}_1\right] + \frac{1}{\Delta\Omega}\int_{\partial S_w} \frac{\partial T_s}{\partial x_i}\cdot \vec{ds}_1 = 0, \tag{73}$$

or for the fluctuating variable

$$\frac{\partial}{\partial x}\left[\frac{1}{\Delta\Omega}\int_{\partial S_w} \hat{T}_s \vec{ds}_1\right] + \frac{1}{\Delta\Omega}\int_{\partial S_w} \frac{\partial \hat{T}_s}{\partial x_i}\cdot \vec{ds}_1 = 0. \tag{74}$$

Travkin and Catton [4,16,17] suggested that the integral heat-transfer terms in Eqs. (71)–(73) be closed in a natural way by III-type heat-transfer law. The second integral term reflects the changing averaged surface temperature along the x coordinate. Eqs. (72) and (73) can be treated using heat-transfer correlations for the heat-exchange integral term (the last term). Regular dilute arrangements of pores, spherical particles or cylinders have been studied much more than random morphologies. Using separate element or 'cell' modeling methods [50,58] to finding the interface temperature field allows one to close the second-'surface' diffusion integral terms in Eqs. (72) and (73) along with Eq. (68).

Many forms of the energy equation are used in the analysis of transport phenomena in porous media. The primary difference between such equations and those resulting from a more rigorous development based on VAT are certain additional terms. The best way to evaluate the need for these additional more complex terms is to obtain an exact mathematical solution and compare the results with calculations using the VAT equations. This will clearly display the need for using the more complex VAT mathematical statements.

Consider a two-phase heterogeneous medium consisting of an isotropic continuous (solid or fluid) matrix and an isotropic discontinuous phase (spherical particles or pores). The volume fraction of matrix, or f-phase, is $\langle m \rangle = m_f = \Delta\Omega_f/\Delta\Omega$, the volume fraction of fillet, or s-phase, is $m_s = 1 - m_f = \Delta\Omega_s/\Delta\Omega$, where $\Delta\Omega = \Delta\Omega_f + \Delta\Omega_s$ is the volume of the REV. The constant properties (phase conductivities, k_f and k_s), stationary (time-independent) heat conduction differential equations for T_f and T_s, the local phase temperatures,

$$k_f \nabla^2 T_f = 0, \quad k_s \nabla^2 T_s = 0,$$

with the IVth kind of interfacial $(f - s)$ thermal boundary conditions. No internal heat sources are present inside the composite sample so that the temperature field is determined by the boundary conditions at the external surface of the sample. After correct formulation of these conditions, the problem is completely stated and has a unique solution.

Two ways to realize a solution to this problem were compared by Travkin and Kushch [60]. The first is the conventional way of replacing the actual composite medium by an equivalent homogeneous medium with an effective thermal conductivity coefficient, $k = k_{eff}(\langle s \rangle, k_f, k_s)$, assuming one knows how to obtain or calculate it. The exact effective thermal coefficient was obtained using DNM based on the mathematical theory of globular morphology multiphase fields developed by Kushch (see, for example [61–63]).

The second way is to solve the problem using the VAT two-equation, three-term integro-differential equations, Eqs. (70) and (73) (see also, for example [4,20]). To evaluate and compare solutions to these equations with the DNM results, one needs to know the local solution characteristics, the averaged characteristics over the both phases in each cell and, in this case, the additional morpho-diffusive terms.

An infinite homogeneous isotropic medium containing a three-dimensional array of spherical particles is chosen for analysis. The particles are arranged so that their centers lie at the nodes of a simple cubic lattice with period a. The temperature field in this heterogeneous medium is caused by a constant heat flux, Q_z, prescribed at the sample boundaries, which, due to the absence of heat sources, leads to the equality of averaged internal heat flux, $\langle q \rangle = Q_z$.

The model composite medium consists of the three regions shown in Fig. 4. The

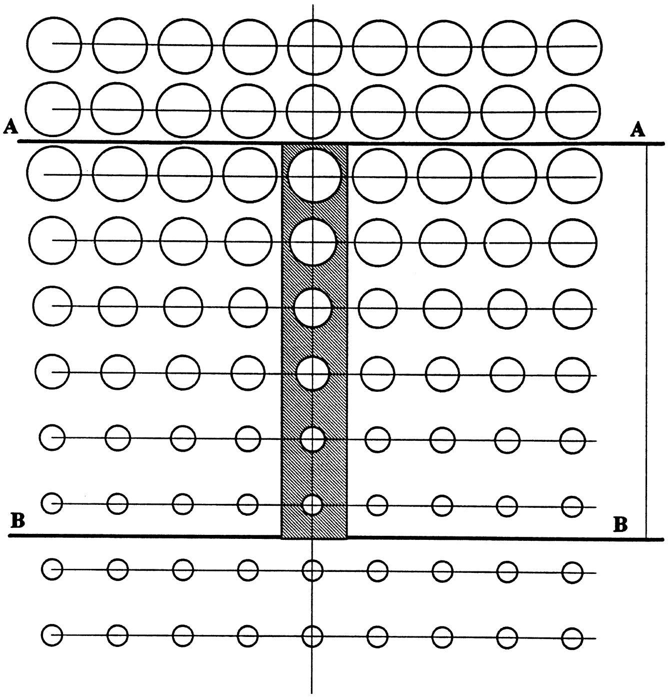

Fig. 4. Model of two-phase medium with variable volume fraction of disperse phase.

half-space lying above the A–A plane has a volume content of the disperse phase $m_s = m_A$ and for half-space below the B–B plane $m_s = m_B$. To define the problem, let $m_A > m_B$. The third part is the composite layer between the plane boundaries A–A and B–B containing N double periodic lattices of spheres (screens) with changing diameters.

The normalized solution of both models (VAT and DNM) for the case of linearly-changing porosity between A–A and B–B and with effective conductivity coefficients of $k_{eff} = 0$, 0.2, 1, 10 and 10 000, respectively, showed practically negligible difference ($< 10^{-3}$) supposedly because of numerical error accumulation [60]. Solutions of the VAT equations, Eqs. (70) and (73), for the composite with varying volume content of disperse phase with accurate DNM closure of the micro model VAT integro-differential terms were obtained implicitly, meaning that each term was calculated independently using the results of DNM calculations.

The coincidence of the results of the exact calculation of the two-equation, three-term energy transport VAT model, Eqs. (70) and (73), with the exact DNM solution and the one-temperature effective coefficient model for heterogeneous media with non-constant spatial morphology clearly demonstrates the need for using all the terms in the VAT equations. The need for the morpho-diffusive terms in the energy equation are further demonstrated by noting that their magnitudes are all of the same order.