

## TRANSPORT PHENOMENA IN HETEROGENEOUS MEDIA: EXPERIMENTAL DATA REDUCTION AND ANALYSIS

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### ABSTRACT

Volume averaging theory (VAT) is used to bring a consistent basis to experimental data reduction and analysis by examples in several areas of transport phenomena. Many common correlations, and their weaknesses, are examined using a unified scaling procedure that allows them to be compared to one another. Momentum resistance and internal heat transfer dependencies are analyzed and compared. VAT based analysis is shown to reveal the influence of morphological characteristics of the media, to suggest scaling parameters that allow a wide variety of different porous media morphologies to be normalized, often eliminating the need for further experimental efforts, and to clarify the relationships between differing experimental configurations. The origin, and insufficiency, of electrical conductivity and momentum transport "cross-correlation" approaches in efforts to find analogies by mathematical comparison without examining the physical foundations of phenomena are described and explained.

### NOMENCLATURE

$\alpha$  - thermal diffusivity [ $m^2/s$ ]  
 $c_d$  - mean drag resistance coefficient in the REV [-]  
 $c_{dp}$  - mean form resistance coefficient in the REV [-]  
 $c_p$  - specific heat [ $J/(kg \cdot K)$ ]  
 $d_{ch}$  - character pore size in the cross section [ $m$ ]  
 $d_h$  - hydraulic diameter [ $m$ ]  
 $d_p$  - particle diameter [ $m$ ]  
 $ds$  - interphase differential area in porous medium [ $m^2$ ]  
 $\partial S_w$  - internal surface in the REV [ $m^2$ ]  
 $f$  - coefficient of hydraulic resistance [-]  
 $F$  - formation factor [-]  
 $\bar{f} \equiv \{f\}_f$  - averaged value  $f$  over  $\Delta\Omega_f$   
 $\langle f \rangle_f$  - value  $f$ , averaged over  $\Delta\Omega_f$  in a REV  
 $\hat{f}$  - morpho-fluctuation value of  $f$  in a  $\Omega_f$   
 $g$  - gravitational constant [ $1/m^2$ ]  
 $h$  - averaged heat transfer coefficient over  $\partial S_w$  [ $W/(m^2K)$ ]  
 $k_{eff}$  - effective thermal conductivity [ $W/(mK)$ ]

$k_f$  - fluid thermal conductivity [ $W/(mK)$ ]  
 $k_s$  - solid phase thermal conductivity [ $W/(mK)$ ]  
 $K$  - permeability [ $m^2$ ]  
 $K_T$  - effective thermal conductivity in fluid phases [ $W/(mK)$ ]  
 $K_m$  - averaged turbulent eddy viscosity [ $m^2/s$ ]  
 $l$  - turbulence mixing length [ $m$ ]  
 $L$  - scale [ $m$ ]  
 $m$  - porosity [-]  
 $\langle m \rangle$  - volume averaged porosity [-]  
 $Nu_{por}$  -  $= \frac{\bar{h}_s d_h}{\lambda_f}$ , interface surface Nusselt number [-]  
 $p$  - pressure [ $Pa$ ] or pitch in regular porous 2D and 3D medium [ $m$ ]  
 $Pe_h$  -  $= Re_h Pr$ , Darcy velocity pore scale Peclet number [-]  
 $Pe_p$  -  $= Re_p Pr$ , particle radius Peclet number [-]  
 $Pr$  -  $= \frac{\nu}{\alpha_f}$ , Prandtl number [-]  
 $Q$  - outward heat flux [ $W/m^2$ ]  
 $q$  - heat flux [ $W/m^2$ ]  
 $Re_{ch}$  - Reynolds number of pore hydraulic diameter [-]  
 $Re_h$  -  $= \frac{\langle m \rangle \bar{u} d_h}{\nu}$ , Darcy velocity Reynolds number of pore hydraulic diameter [-]  
 $Re_p$  -  $= \frac{\bar{u} d_p}{\nu}$ , particle Reynolds number [-]  
 $Re_{por}$  -  $= \frac{\bar{u} d_{por}}{\nu}$ , Reynolds number of general scale pore hydraulic diameter [-]  
 $St$  - Stanton Number [-]  
 $S_w$  - specific surface of a porous medium  $\partial S_w / \Delta\Omega$  [ $1/m$ ]  
 $S_{wp}$  [ $1/m$ ] =  $S_L / \Delta\Omega$   
 $T_s$  - solid phase temperature [ $K$ ]  
 $u$  - velocity [ $m/s$ ]  
 $U$  - velocity in x-direction [ $m/s$ ]

### SUBSCRIPTS

$D$  - Darcy  
 $e$  - effective  
 $f$  - fluid phase  
 $h$  - hydraulic  
 $i$  - component of vector variable or species  
 $k$  - component of turbulent variable that designates turbulent "microeffects" on a pore level

*L* - laminar  
*m* - scale value  
*p* - particle  
*por* - porous  
*r* - roughness  
*s* - solid phase  
*T* - turbulent

## SUPERSCRIPTS

$\sim$  - value in fluid phase averaged over the REV  
 $\bar{\phantom{x}}$  - mean turbulent quantity  
 $\prime$  - turbulent fluctuation value  
 $\circ$  - value in solid phase averaged over the REV

## GREEK LETTERS

$\tilde{\alpha}$  - averaged heat transfer coefficient over  $\partial S_w$  [ $W/(m^2K)$ ]  
 $\Delta\Omega$  - representative elementary volume (REV) [ $m^3$ ]  
 $\Delta\Omega_f$  - pore volume in a REV [ $m^3$ ]  
 $\Delta\Omega_s$  - solid phase volume in a REV [ $m^3$ ]  
 $\Phi$  - potential function  
 $\sigma$  - effective conductivity tensor  
 $\lambda_f$  - fluid thermal conductivity [ $W/(mK)$ ]  
 $\mu$  - dynamic viscosity [ $kg/(ms)$ ] or [ $Pa\cdot s$ ]  
 $\nu$  - kinematic viscosity [ $m^2/s$ ]  
 $\rho$  - density [ $kg/m^3$ ]

## 1. INTRODUCTION

Among the important features of volume averaging theory (VAT) are that it allows specific medium types and morphologies, lower-scale fluctuations of variables, cross-effects of different variable fluctuations, and interface variable fluctuations effects, etc. to be considered. It is not possible to include all of these characteristics in current models using conventional theoretical approaches. At present, effective coefficients are measured in what are often inappropriate ways considering their usage because there are only approximate descriptions of the physical phenomena and often the description is simply wrong. Frequently the assumption is made that the heterogeneous media is homogeneous and then the fields are described by classical homogeneous differential equations using the inappropriately measured effective transport coefficients.

The design and optimization of hierarchical materials using VAT based models allows one to connect properties, morphologies and other features. For example, experimental search for improved piezoelectric materials is usually done by adding a 3rd and/or 4th component into the piezoelectric material and measuring the result. This can be done using the VAT based equations in a much more direct way because the effects caused by adding the additional components and, of course, by the morphology of the 4th component are more correctly represented by the models.

The use of VAT in developing models that can be used for data evaluation and extrapolation are that it allows consideration of

- 1) the effects of interfaces and grain boundaries,
- 2) the effects of the morphology of the phases by incorporating the physical field equations,
- 3) a rigorous mathematical description of separate and combined fields and their interactions without incorporation of effective coefficients,
- 4) a correct mathematical description of macroscale effective transport coefficients including their dependencies on microscale parameters,
- 5) a correct analysis of experimental data from heterogeneous media experiments, and
- 6) a method of determining the effect of additional components in a piezoelectric material.

To demonstrate the benefits of using VAT based models in experimental data analysis, four examples of transport phenomena are chosen; 1) momentum transport (friction), 2) heat transfer, , thermal conduction and electrical conductivity for DC and AC electrical fields. The use of VAT based models of heterogeneous media is the same for the four different examples and many of the equations are similar.

## 2. EXPERIMENTAL ASSESSMENT OF FLOW RESISTANCE IN A POROUS MEDIUM

The one term flow resistance model for porous media often used for experimental data analysis is

$$-\frac{d\tilde{p}}{dx} = f \left( \frac{S_w}{\langle m \rangle} \right) \frac{\rho_f \tilde{u}^2}{2}, \quad (1)$$

where  $f$  is a coefficient of hydraulic resistance. The two-term models used for flow resistance experimental data reduction having first and second order velocity terms are often called Darcy-Forchheimer flow resistance models. These models allow direct comparison to be made with empirical and semi-empirical relationships like the Darcy and Darcy-Forchheimer resistance models. The momentum equation for laminar as well as for high flow regimes often used is the model by Ergun (1952),

$$-\frac{d\tilde{p}}{dx} = \frac{\mu}{K_D} \langle m \rangle \tilde{u} + \rho_f A \langle m \rangle^2 \tilde{u}^2. \quad (2)$$

A model by Vafai and Kim (1989) for the same flow range is

$$-\frac{d\tilde{p}}{dx} = \frac{\mu}{K_D} \langle m \rangle \tilde{u} + \rho_f \langle m \rangle^3 \frac{F}{K_D^{\frac{1}{2}}} \tilde{u}^2, \quad (3)$$

and a model by Poulikakos and Renken (1987) is

$$-\frac{d\tilde{p}}{dx} = \frac{\mu}{K_D} \tilde{u} + \rho_f A \tilde{u}^2. \quad (4)$$

Analysis of media where exact calculation of friction factors can be made show that the Darcy, Darcy-Forchheimer or Ergun type model correlations are neither consistent for any regime nor have a correct theoretical foundation. An ideal parallel tube morphology model yields the following Darcy friction coefficient model

$$\frac{\Delta p}{L} = f_D \frac{\rho_f \tilde{U}^2}{d_h}, \quad f_D = \frac{2d_h}{\rho_f \tilde{U}^2} \frac{\Delta p}{L}, \quad (5)$$

For an equal diameter straight tube morphology, the morphology function,  $S_w / \langle m \rangle$ , is found from

$$S_w = \frac{\partial S_w}{\partial \Omega} = \frac{2\pi R}{p \Delta y}, \quad \langle m \rangle = \frac{\pi R^2}{p \Delta y}, \quad \frac{S_w}{\langle m \rangle} = \frac{4}{d_h}, \quad (6)$$

For this morphology the Fanning friction factor (using 6) is related to the Darcy friction factor by

$$\frac{\Delta p}{L} = \frac{f_D}{4} \left( \frac{4}{d_h} \right) \frac{\rho_f \tilde{U}^2}{2} = \left( \frac{f_D}{4} \right) \left( \frac{S_w}{\langle m \rangle} \right) \frac{\rho_f \tilde{U}^2}{2}, \quad (7)$$

$$f_f = \frac{d_h}{2\rho_f \tilde{U}^2} \frac{\Delta p}{L}, \quad (8)$$

from which one obtains (Travkin and Catton, 1995), with  $f_f = c_d$  for this geometry,

$$f_f = \frac{f_D}{4} = c_d. \quad (9)$$

The friction coefficient  $c_d$  for smooth tubes is often calculated using the Nikuradze and Blasius formulae (Schlichting, 1979).

The flow resistance models are written in terms of the second power of the bulk velocity variable. Convergence of the VAT based flow resistance transport models to these classical constructions has been demonstrated on several occasions by Travkin and Catton (1992, 1995, 1998, 1999).

Exact flow resistance results obtained using the VAT based governing equations by Travkin and Catton (1995, 1998a, 1999b) for a random pore diameter distribution for almost the same morphology as used by Achdou and Avellaneda (1992) yielded a wide departure from Darcy law based treatments. This was shown even for single pore morphology with one diameter different from the others. Using consistent VAT based procedures (Travkin and Catton, 1999b) one can easily find the variable, nonlinear permeability coefficient for use with Darcy flow to be

$$K_{dc} = \left[ f_f \left( \frac{S_w}{\langle m \rangle^2} \right) \left( \frac{\tilde{U}}{2\nu} \right) \right]^{-1}. \quad (10)$$

based on exact analytical (in laminar regime) or well established correlations for the Fanning friction factor in tubes.

### 3. MOMENTUM RESISTANCE IN 1D MEMBRANE AND POROUS LAYER TRANSPORT VAT EQUATIONS

The steady state VAT based one dimensional governing equations in a porous media for laminar transport (Travkin and Catton, 1998) are the momentum equation

$$\begin{aligned} \langle m \rangle \tilde{U} \frac{\partial \tilde{U}}{\partial x} + \frac{\partial}{\partial x} \langle \tilde{u} \tilde{u} \rangle_f + \frac{1}{\rho_f} \frac{\partial}{\partial x} \left( \langle m \rangle \{p\}_f \right) = \\ - \frac{1}{\rho_f \Delta \Omega} \int_{\partial S_w} p \vec{ds} \\ + \nu \frac{\partial}{\partial x} \left( \frac{\partial \langle m \rangle \tilde{U}}{\partial x} \right) + \frac{\nu}{\Delta \Omega} \int_{\partial S_w} \frac{\partial \tilde{U}}{\partial x_i} \cdot \vec{ds}, \end{aligned} \quad (11)$$

and the energy equations for fluid and solid phases

$$\begin{aligned} c_{pf} \rho_f \langle m \rangle \tilde{U} \frac{\partial \tilde{T}_f}{\partial x} \\ = k_f \frac{\partial}{\partial x} \left( \frac{\partial \langle m \rangle \tilde{T}_f}{\partial x} \right) + c_{pf} \rho_f \frac{\partial}{\partial x} \left( \langle m \rangle \{ -\tilde{T}_f \tilde{u} \}_f \right) \end{aligned} \quad (12)$$

$$+ \frac{\partial}{\partial x} \left( \frac{k_f}{\Delta \Omega} \int_{\partial S_w} T_f \vec{ds} \right) + \frac{1}{\Delta \Omega} \int_{\partial S_w} k_f \frac{\partial T_f}{\partial x_i} \cdot \vec{ds},$$

$$\begin{aligned} \frac{\partial}{\partial x} \left( \frac{\partial (1 - \langle m \rangle) \{T_s\}_s}{\partial x} \right) + \frac{\partial}{\partial x} \left( \frac{1}{\Delta \Omega} \int_{\partial S_w} T_s \vec{ds}_1 \right) \\ = - \frac{1}{\Delta \Omega} \int_{\partial S_w} \frac{\partial T_s}{\partial x_i} \cdot \vec{ds}_1 \end{aligned} \quad (13)$$

The momentum equation for turbulent flow of an incompressible fluid in a porous medium based on the  $k$ -theory (Travkin and Catton, 1992, 1995) is of the form

$$\begin{aligned} \langle m \rangle \left( \frac{\partial \tilde{U}}{\partial t} + \tilde{U} \frac{\partial \tilde{U}}{\partial x} \right) = \\ \frac{1}{\Delta \Omega} \int_{\partial S_w} (K_m + \nu) \frac{\partial \tilde{U}}{\partial x_i} \cdot \vec{ds} + \frac{\partial}{\partial x} \left[ \langle m \rangle \left( \tilde{K}_m + \nu \right) \frac{\partial \tilde{U}}{\partial x} \right] + \\ \frac{\partial}{\partial x} \left( \langle m \rangle \left\{ \tilde{K}_m \frac{\partial \tilde{u}}{\partial x} \right\}_f \right) + \frac{\partial}{\partial x} \left( \langle m \rangle \left\{ -\tilde{u} \tilde{u} \right\}_f \right) - \\ \frac{1}{\rho_f \Delta \Omega} \int_{\partial S_w} \tilde{p} \vec{ds} - \frac{1}{\rho_f} \frac{\partial}{\partial x} \left( \langle m \rangle \tilde{p} \right). \end{aligned} \quad (14)$$

Other models for momentum transport in porous media were analyzed by Travkin and Catton (1992, 1995, 1998). It can be clearly shown that there are major differences between these equations and the conventionally mathematical models.

For laminar regime when the porosity is constant the above equation can be simplified as

$$\begin{aligned} \frac{\partial}{\partial x} (\langle m \rangle \tilde{p}) + \frac{1}{\Delta \Omega} \int_{\partial S_w} p \vec{ds} + \rho_f u_{*rk}^2 S_w(x) = \\ - \langle m \rangle \rho_f \tilde{U} \frac{\partial \tilde{U}}{\partial x} + \mu \frac{\partial}{\partial x} \left[ \frac{\partial \langle m \rangle \tilde{U}}{\partial x} \right] + \\ + \frac{\partial}{\partial x} \left[ \rho_f \langle m \rangle \{ -\hat{u} \hat{u} \}_f \right]. \end{aligned} \quad (15)$$

The left part of equation (15) is the equation used in almost all experimental work addressing pressure loss in porous media. Correlation of experimental data using a correlation like

$$-\frac{d \tilde{p}}{dx} = f_f \left( \frac{S_w}{\langle m \rangle} \right) \frac{\rho_f \tilde{U}^2}{2}, \quad (16)$$

implicitly includes the R.H.S. of equation (15). Friction factor data presented in this way loses generality because it does not reflect the right hand three terms from equation (15) (Travkin and Catton, 1995). The correlating equation should be written as

$$-\frac{d(\langle m \rangle \tilde{p})}{dx} = \left( c_f + c_{dp} \frac{S_{wp}}{S_w} + R_m \right) (S_w(x)) \frac{\rho_f \tilde{U}^2}{2}, \quad (17)$$

where  $R_m$  is deduced from the following relationship

$$\begin{aligned} R_m \left( S_w(x) \frac{\rho_f \tilde{U}^2}{2} \right) = \\ = \langle m \rangle \rho_f \tilde{U} \frac{\partial \tilde{U}}{\partial x} + \rho_f \frac{\partial}{\partial x} \left[ \langle m \rangle \{ \hat{u} \hat{u} \}_f \right] - \\ - \mu \frac{\partial}{\partial x} \left[ \frac{\partial \langle m \rangle \tilde{U}}{\partial x} \right]. \end{aligned} \quad (18)$$

In a porous medium experimental sample with constant porosity and steady flow and far from an inlet or an exit, one can assume that all four terms on the right hand side of (15) are equal to zero. Unfortunately, a tremendous amount of experimental data are produced under conditions which the previous simplifying assumptions can not be made and the impact of  $R_m$  is included in the sought after friction factor. If one wants to distinguish the effects of input-output pressure loss from viscous friction and drag resistance components inside of porous medium, the need for accounting for the terms (18) is clear.

More interesting conditions exist in high energy rate exchange regenerators where flow and temperature regimes inside the regenerator are transient and steeply inhomogeneous in space. The inhomogeneity in space and time precludes neglecting the four right hand terms in equation (15). Inhomogeneous terms on the right hand side of (18) may be analyzed using the scaling technique and some can easily be

interpreted. For example, the first right hand term is the convective term which can have a strong dependency on thickness of a porous specimen. As a result many studies report a data correlation with specimen thickness. The next terms are the "morpho-convective" term and the momentum diffusion term. The morpho-convective term is the result of morphology induced fluctuations and is neglected in most models.

It is a difficult task to properly evaluate flow resistance models because of the multivariant character of all the possible experimental set-ups. We can, however, bring some order to the process by beginning to measure and model these terms.

In order to test some VAT mathematical models and the closure models we constructed Laser Doppler Velocimetry (LDV) has been used to measure the velocity field and friction factor for flow of air in a duct with regular rib roughness. The instantaneous turbulent velocity were measured in a rough rectangular duct of which the top and bottom were mounted with 6.35 mm x 6.35 mm ribs with a spacing of 75 mm. So in the rough layer, the porosity is  $\langle m \rangle = 6.35/75.0 = 0.0847$  and in the middle of the duct the porosity is unity. The numerical results of the above models presented in Travkin et al. (1998) compare well with the experimental results presented in Huckle et al. (1999). The data generated from this experiment can serve as a foundation upon which future closure modeling may lie.

## SIMULATION PROCEDURES

A procedure for analysis of data from different sources was developed by Travkin and Catton (1998) that allows comparison of different porous media morphology and length scales based on the porous media Reynolds number  $Re_{por}$ . Some of the models and correlations compared are the following:

1) Gortyshov et al. (1987) correlations were experimentally derived for the Reynolds-Forchheimer momentum equation in the form

$$\alpha = 6.61 \cdot 10^7 (\bar{d}_h)^{-1.98} < m >^{(-4.75)}, \quad (19)$$

$$\beta = 5.16 \cdot 10^2 (\bar{d}_h)^{-1.07} < m >^{(-11.16)}, \quad (20)$$

where the hydraulic diameter  $\bar{d}_h$  is taken in millimeters

$$\bar{d}_h = \frac{d_h[m]}{0.001[m]}. \quad (21)$$

3) Beavers and Sparrow (1969) developed the following expression

$$f_f(Re_{por}) = \left( \frac{1}{Re_{por}} \left( \frac{4\sqrt{\alpha}}{S_w} \right) + 0.074 \right) \left( \frac{2\sqrt{\alpha} < m >^3}{S_w} \right). \quad (22)$$

The experimental correlations are reformulated for comparison with the simplified VAT 1D momentum equation

(which can include two, three or five flow resistance mechanisms)

$$\frac{\Delta P}{L} = f_f (Re_{por}) \left( \frac{S_w}{\langle m \rangle} \right) \frac{\rho_f \tilde{U}^2}{2}, \quad (23)$$

then the Fanning friction factor correlation can be easily compared and analyzed. Some correlations are presented in Travkin and Catton (1998) using the scaling procedures suggested above.

#### 4. INTERNAL HEAT TRANSFER COEFFICIENT MODELING

The VAT based approach used to analyze and compare data for momentum resistance is used for the effective internal heat transfer coefficient in porous medium. As was noted by Viskanta (1995), "Convective heat and mass transfer in consolidated porous materials has received practically no theoretical research attention. This is partially due to the complexity which arises as a result of physical and chemical heterogeneity that is difficult to characterize with the limited amount of data that can be obtained through experiments." Again we will use scaling to normalize the data and to reduce some of the complexity.

Viskanta (1995) generalized measurements of the internal porous ceramic heat transfer coefficients to the form

$$Nu_v = 2.0 + a Re^b Pr^{\frac{1}{3}}, \quad (24)$$

assuming that the Nusselt number should limit to 2.0 when the Re number decreases to zero. This assumption is only justified for unconsolidated sparse spherical particle morphologies and is questionable for other porous medium morphologies, especially consolidated media. This is why some researchers ignore this limit when correlating their findings.

#### SIMULATION PROCEDURES

A large amount of the data provided by Viskanta (1995a,b) were used to deduce a few comparable correlations internal porous medium heat transfer characteristics. The VAT based scaling approach described above for the flow resistance in porous media was used. Some of the work the VAT based scaling was applied to are summarized below.

- 1) Kar and Dybbs (1982) report

$$Nu_p = \frac{\tilde{h}_s d_p}{\lambda_f} = 0.004 Re_p^{1.35} Pr^{\frac{1}{3}}, \quad (25)$$

where

$$Re_p = \frac{\tilde{U} d_p}{\nu}, \quad (26)$$

with possible (for particulate media) equality

$$d_p = 3(1 - \langle m \rangle) / 2 \langle m \rangle. \quad (27)$$

If the hydraulic diameter  $d_h$  is

$$d_h \cong d_{por} = \frac{4 \langle m \rangle}{S_w}, \quad (28)$$

then the particle Reynolds number  $Re_p$  can be rewritten using (27)

$$Re_p = Re_{por} \left( \frac{3(1 - \langle m \rangle)}{2 \langle m \rangle} \right), \quad (29)$$

and used in the above correlation (25). To one needs to relate  $d_p$  to  $d_h$  by assuming the latter is equal to (28), then

$$\begin{aligned} Nu_{por} &\cong \frac{\tilde{h}_s d_h}{\lambda_f} = Nu_h = \frac{2 \langle m \rangle}{3(1 - \langle m \rangle)} Nu_p (Re_p) = \\ &= \frac{2 \langle m \rangle}{3(1 - \langle m \rangle)} Nu_p \left( \left( \frac{3(1 - \langle m \rangle)}{2 \langle m \rangle} \right) Re_{por} \right) \end{aligned} \quad (30)$$

- 2) Achenbach (1995) found

$$Nu_h = \left\{ (1.18 Re_h^{0.58})^4 + \left[ 0.23 \left( \frac{Re_h}{\langle m \rangle} \right)^{0.75} \right]^4 \right\}^{\frac{1}{4}}, \quad (31)$$

for  $Pr = 0.71$ ,  $\langle m \rangle = 0.387$ ,  $1 < (Re_h / \langle m \rangle) < 7.7 \cdot 10^5$ . The hydraulic diameter Reynolds number and the pore Nusselt number are related to the pore Reynolds number and pore Nusselt number by

$$Re_h \cong Re_{por} \langle m \rangle,$$

and

$$Nu_{por} (Re_{por}) \cong Nu_h (Re_{por} \langle m \rangle). \quad (32)$$

- 3) Younis and Viskanta (1993a,b) obtained a correlation for consolidated cellular ceramics

$$Nu_{vh} = \frac{\tilde{h}_v d_h^2}{\lambda_f} = \left( 0.0098 + 0.11 \left( \frac{d_h}{L} \right) \right) Re_h^{1.3} Pr^{\frac{1}{3}}, \quad (33)$$

where  $\langle m \rangle = 0.83 \div 0.87$ . Here the  $Nu_v$  increases when the test specimen thickness decreases. Perhaps this is the influence of the boundary momentum and energy resistance.  $Nu_v$  is related to  $Nu_{por}$  by

$$Nu_{por} = \frac{Nu_{vh} (Re_{por} \langle m \rangle)}{4 \langle m \rangle}. \quad (34)$$

- 4) The Kays and London (1984) relation is

$$StPr^{(2/3)} = 1.4Re_{por}^{-0.45}, \quad (35)$$

which transforms to

$$\frac{NuPr^{(2/3)}}{Re_{por}Pr} = 1.4Re_{por}^{-0.45}, \implies Nu_{por} = 1.4Re_{por}^{0.55}Pr^{1/3}. \quad (36)$$

Some useful observations can be made by comparing the heat transfer predictions shown in Fig. 1. One of the most significant observations is that the magnitude of the differences among the correlations of Kar and Dybbs (1982), Younis and Viskanta (1993), Rajkumar (1993) and others cannot be explained without accounting for specifics of the media, the design of the experiment and treatment of the data. Similarly, the coincidence of the correlations by Kays and London (1984), Achenbach (1995), and Kokorev et al. (1987) are a surprise as they were developed using different techniques and basic approaches. The data reduction given in the Heat Exchangers Design Handbook (1983) reflects a highly tuned correlation in the low Reynolds number range. That is why transformation of this correlation cannot be based on one specific approach (as, for example, globular morphology with specific globular diameter). An obvious transformation from particle to pore scale using general simple technique as described above doesn't work properly in this case.

## 5. THERMAL CONDUCTIVITY MEASUREMENTS IN TWO-PHASE MEDIUM

Many thermal conduction experiments are based on a constant heat flux through the experimental specimen with data reduction using (see, for example, Uher, 1990)

$$K = \frac{QL}{A\Delta T}, \quad (37)$$

where  $Q$  is the electrical power from heater dissipated through the specimen,  $L$  is the distance used to measure the temperature difference,  $A$  is the uniform cross-sectional area of the sample. This overly simplified approach will be shown to be inadequate for heterogeneous media.

### VAT BASED CONSIDERATIONS FOR EXPERIMENTAL DATA REDUCTION

#### CONSTANT HEAT CONDUCTIVITY COEFFICIENT

To demonstrate the problems with an experiment to determine the thermal conductivity of phase 1 of a composite material or a material that is composite but thought to be a pure substance we start with the VAT bulk diffusion term in

the energy equation

$$k_1 \nabla^2 (\langle s_1 \rangle \tilde{T}_1) + k_1 \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] = \nabla \cdot [-\vec{q}_1], \quad (38)$$

where the "diffusive" like flux  $\vec{q}_1$  contains one more than that is conventionally considered

$$\begin{aligned} \vec{q}_1 &= -k_{eff,1} \nabla (\langle s_1 \rangle \tilde{T}_1) \\ &= -k_1 \left( \nabla (\langle s_1 \rangle \tilde{T}_1) + \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right). \end{aligned} \quad (39)$$

The heat flux in phase 1 is determined from the averaged temperature  $\tilde{T}_1$ , resulting in the effective (not homogeneous) conductivity coefficient in phase 1 being defined by

$$\begin{aligned} k_{eff,1} &= k_1 \left[ \nabla (\langle s_1 \rangle \tilde{T}_1) + \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] (\nabla (\langle s_1 \rangle \tilde{T}_1))^{-1} \\ &= k_1 \left[ 1 + \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 / (\nabla (\langle s_1 \rangle \tilde{T}_1)) \right]. \end{aligned} \quad (40)$$

After these transformations the heat transfer equation in phase 1 becomes

$$\langle s_1 \rangle (\rho c_p)_1 \frac{\partial \tilde{T}_1}{\partial t} = \nabla \cdot [k_{eff,1} \nabla (\langle s_1 \rangle \tilde{T}_1)] + \tilde{\alpha}_{21} S_{12} (\{T\}_2 - \{T\}_1). \quad (41)$$

The last term on the R.H.S. terms is the result of the closure consideration

$$\begin{aligned} \frac{1}{\Delta \Omega} \int_{\partial S_{12}} k_1 \frac{\partial T}{\partial x_i} \cdot \vec{d}s_1 &= -\frac{1}{\Delta \Omega} \int_{\partial S_{12}} k_1 \frac{\partial T}{\partial \mathbf{n}_2} \cdot \mathbf{n}_2 = \\ &= \frac{1}{\Delta \Omega} \int_{\partial S_{12}} \mathbf{q}_2 \cdot \vec{d}s_2 = \tilde{\alpha}_{21} S_{12} (\{T\}_2 - \{T\}_1), \end{aligned} \quad (42)$$

where the closure procedure is applicable to description of the fluid-solid media heat exchange as well as for the case of solid-solid heat exchange as done in many works. A more strict and precise integration of the heat flux over the interface surface gives exact closure for the term in the governing equations for the neighboring phases. Note that  $k_{eff,1}$  does not depend explicitly on the phase heat conductivity coefficient  $k_1$ . Generally speaking it depends on  $k_1$  implicitly through the boundary conditions and the conditions at the interface surface  $\partial S_{12}$ . In the more complex and challenging physics of interface transport modeling, treating the interface as a third phase, the third interphase exchange term and the second term are issues closely connected to the closure problem and to models of interface surface transport.

When an experimentalist evaluates experimental data using the equation

$$(\rho c_p)_1 \frac{\partial \tilde{T}_1}{\partial t} = \nabla \cdot [k_1 \nabla \tilde{T}_1], \quad (43)$$

for determination of the phase 1 effective conductivity, two mistakes are made:

- 1) the material homogeneous conductivity coefficient  $k_1$  (which is the subject of experiment) is confused with the effective coefficient  $k_{eff,1}$  of the same phase in a composite, another variable, and
- 2) carrying out data reduction using this equation for the modeling equation equates

$$k_{eff,1} \cong k_1, \quad (44)$$

losing the term reflecting the interface exchange rate

$$\tilde{\alpha}_{21} S_{12} (\{T\}_2 - \{T\}_1), \quad (45)$$

that in a composite material will depend on at least two temperatures and could have a large influence on the internal exchange rate (see works by Travkin and Kushch, 1999a,b; and Travkin and Catton, 1998b).

#### COMPOSITE MATERIAL'S BULK HEAT CONDUCTIVITY COEFFICIENTS

The problem becomes difficult in the case when effective conductivity coefficient meant as for the whole composite material. Combining both temperature equations (if only two of phases are present) for the simplest case of constant coefficients keeping in mind that the multiphase averaged temperature is

which has the three different temperatures  $\tilde{T}_1$ ,  $\tilde{T}_2$ , and  $T_1$  ( $\partial S_{12}$ ) (here  $\langle T \rangle = \langle s_1 \rangle \tilde{T}_1 + \langle s_2 \rangle \tilde{T}_2$ ).

And only assuming a local thermal equilibrium

$$\langle T \rangle = \langle s_1 \rangle \tilde{T}_1 + \langle s_2 \rangle \tilde{T}_2 = T^* = \tilde{T}_1 = \tilde{T}_2, \quad (46)$$

the mixed temperature equation becomes two-temperature  $T^*$ ,  $T_1$  ( $\partial S_{12}$ ) dependable with simplified left hand part of the equation

$$\begin{aligned} & (\langle s_1 \rangle (\rho c_p)_1 + \langle s_2 \rangle (\rho c_p)_2) \frac{\partial T^*}{\partial t} \\ &= \nabla \cdot [(k_1 \nabla (\langle s_1 \rangle T^*) + k_2 \nabla (\langle s_2 \rangle T^*))] \\ &+ (k_1 - k_2) \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right], \end{aligned} \quad (47)$$

with the two different temperatures the effective coefficient of conductivity equals to

$$k_{eff}^* = \{[(k_1 \nabla (\langle s_1 \rangle T^*) + k_2 \nabla (\langle s_2 \rangle T^*))]\} \quad (48)$$

$$+ (k_1 - k_2) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] \} (\langle \nabla T^* \rangle)^{-1}.$$

When equilibrium cannot be assumed, the derivation gets more complex and the transient effective nonequilibrium coefficient in a two phase medium is found to be

$$\begin{aligned} k_{eff}^o &= a_{eff}^o (\langle s_1 \rangle (\rho c_p)_1 + \langle s_2 \rangle (\rho c_p)_2) = \\ &= (\langle s_1 \rangle (\rho c_p)_1 + \langle s_2 \rangle (\rho c_p)_2) \left\{ a_1 \nabla (\langle s_1 \rangle \tilde{T}_1) + a_2 \nabla (\langle s_2 \rangle \tilde{T}_2) \right\} \\ &+ (a_1 - a_2) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] + \mathbf{A} \} (\langle \nabla T \rangle)^{-1}, \end{aligned} \quad (49)$$

where

$$\mathbf{A} = \nabla^{(-1)} \left\{ a_1 \left( 1 - \frac{a_2 k_1}{a_1 k_2} \right) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_1 \cdot \vec{d}s_1 \right] \right\}. \quad (50)$$

This is a complex expression for analytical, experimental or numerical assessment. The solution of this problem, which includes finding the effective bulk composite material heat conductivity coefficient, is equivalent to a solution of the exact two-phase problem. Comparing expressions for the transient coefficient (49) and the thermal equilibrium coefficient (48) show that they are very different both in definition and in calculation. It does not matter what kind of mathematical statement is used for the problem, two separate heat transfer equations or the VAT statement, the problem complexity is the same. The benefit of the VAT based equations is a correct estimation of the transient effective conductivity.

## 6. EFFECTIVE ELECTRICAL CONDUCTIVITY IN HETEROGENEOUS MEDIA

### EFFECTIVE ELECTRICAL CONDUCTIVITY MODELING

An expression for the effective electrical conductivity is found starting with the conventional constant phase conductivity equations for each phase of the heterogenous medium. Following the same procedure as outlined for thermal conductivity, the VAT equation for the potential in phase 1 is

$$\nabla^2 (\langle m_1 \rangle \tilde{\Phi}_1) + \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} \Phi_1 \vec{d}s_1 \right] + \frac{1}{\Delta \Omega} \int_{\partial S_{12}} \nabla \Phi_1 \cdot \vec{d}s_1 = 0, \quad (51)$$

which is the same as the equation for the temperature field in phase 1.

The standard definition of an effective (macroscopic) conductivity tensor results from the following equation

$$\langle \mathbf{j} \rangle = -\sigma_{ij}^* \langle \nabla \Phi \rangle, \quad (52)$$

where it is assumed that

$$\begin{aligned}
\langle \mathbf{j} \rangle &= \langle \mathbf{j} \rangle_1 + \langle \mathbf{j} \rangle_2 = -\sigma_1 \langle \nabla \Phi \rangle_1 - \sigma_2 \langle \nabla \Phi \rangle_2 \quad (53) \\
&= -\sigma_{ij}^* \langle \nabla \Phi \rangle = -\sigma_{ij}^* \nabla \langle \Phi \rangle \\
&= -\sigma_{ij}^* [\langle \nabla \Phi \rangle_1 + \langle \nabla \Phi \rangle_2] \\
&= -\sigma_{ij}^* \langle \nabla \Phi \rangle_1 - \sigma_{ij}^* \langle \nabla \Phi \rangle_2,
\end{aligned}$$

resulting in an expression for the effective coefficient

$$\begin{aligned}
\sigma_{ij}^* &= \left[ \sigma_1 \nabla \left( \langle m_1 \rangle \tilde{\Phi}_1 \right) + \sigma_2 \nabla \left( \langle m_2 \rangle \tilde{\Phi}_2 \right) \right. \\
&\quad \left. + (\sigma_2 - \sigma_1) \frac{1}{\Delta \Omega} \int_{\partial S_{12}} \Phi_2 \vec{d}s_2 \right] \\
&\quad / [\langle \nabla \Phi \rangle_1 + \langle \nabla \Phi \rangle_2]. \quad (54)
\end{aligned}$$

that involves knowledge of three potential functions  $\tilde{\Phi}_1$ ,  $\tilde{\Phi}_2$ ,  $\Phi_{1|\partial S_{12}}$  in the volume  $\Omega$ . The second term on the R.H.S. again emphasizes the importance of the phase interfaces. This expression for the steady state effective conductivity can be shown to be equal to the known expression

$$\begin{aligned}
\sigma_{ij}^* \nabla \langle \Phi \rangle &= \sigma_2 \nabla \langle \Phi \rangle + (\sigma_1 - \sigma_2) \frac{1}{\Delta \Omega} \int_{\Delta \Omega_1} \nabla \Phi \, d\omega = \\
&= \sigma_2 \nabla \langle \Phi \rangle + (\sigma_1 - \sigma_2) \langle \nabla \Phi \rangle_1. \quad (55)
\end{aligned}$$

The same conclusions can be drawn about the potential in both phases as was done above for temperature fields.

Experimental evidence of the importance of the interface surface, a natural part of any VAT based model, to heterogeneous media modeling is shown in the figures presented in Travkin et al. (1999). The figures showed different types of structures of polypropylene (PP) based composites and the methods of their synthesis, as well as critical indices  $t$  of percolation theory, which defines concentration dependence of conductivity by  $\sigma = \sigma_f (v_f - v_f^*)^t$ , where  $v_f$  is the concentration of conducting filler and  $v_f^*$  is the percolation threshold.

#### CROSS-CHARACTERISTICS MODELING IN TERMS OF VAT

The mathematical similarities between electrical potential fields and creeping velocity fields has led to development of similar expressions for flow permeability and electrical conductivity in a fluid phase located in an insulating porous solid phase. Johnson et al. (1986) used this view to suggest that the dependency for fluid static permeability in porous media can be written as

$$k_{dc} \approx \frac{\Lambda^2}{8F}, \quad (56)$$

where  $\Lambda$  is a parameter that relates pore volume -to- surface in a porous medium using the known electrical field  $\mathbf{E}(\mathbf{r})$

which is in turn found through an electrical potential  $\mathbf{E} = -\nabla \Phi$ .

The way in which the electrical field quadratic absolute function appeared in volumetric and surface integrals in Johnson et al. (1986) will be described in more detail. First, consider the transformation of the main conductivity equation, assumed to be a homogeneous medium, in terms of the factor  $\Phi(\mathbf{r})$

$$\begin{aligned}
&\nabla \cdot [\Phi(\mathbf{r}) (\sigma(\mathbf{r}) \nabla \Phi(\mathbf{r}))] \\
&= \sigma(\mathbf{r}) |\nabla \Phi(\mathbf{r})|^2 + \Phi(\mathbf{r}) \nabla \cdot (\sigma(\mathbf{r}) \nabla \Phi(\mathbf{r})) \\
&= \sigma(\mathbf{r}) |\nabla \Phi(\mathbf{r})|^2. \quad (57)
\end{aligned}$$

If this identity is subject to integration over the fluid volume to which the voltage drop  $\Phi_L$  is applied, then the Gauss-Ostrogradsky theorem of divergence for effective conductivity will be justified following the transformation

$$\begin{aligned}
\int_{\Omega_f} \nabla \cdot [\Phi(\mathbf{r}) (\sigma(\mathbf{r}) \nabla \Phi(\mathbf{r}))] \, d\omega &= \int_{\partial S_f} [\Phi(\mathbf{r}) (\sigma(\mathbf{r}) \nabla \Phi(\mathbf{r}))] \cdot \vec{d}s_1 = \\
&= \frac{1}{2} \int_{\partial S_f} [(\sigma(\mathbf{r}) \nabla \Phi^2(\mathbf{r}))] \cdot \vec{d}s_1 = -\Phi_L \mathbf{j} = \int_{\Omega_f} \sigma(\mathbf{r}) |\nabla \Phi(\mathbf{r})|^2 \, d\omega, \quad (58)
\end{aligned}$$

The left hand side of the equation should be

$$\begin{aligned}
\int_{\Omega_f} \nabla \cdot [\Phi(\mathbf{r}) (\sigma(\mathbf{r}) \nabla \Phi(\mathbf{r}))] \, d\omega &= \int_{\partial S_{12}} [\Phi(\mathbf{r}) (\sigma(\mathbf{r}) \nabla \Phi(\mathbf{r}))] \cdot \vec{d}s_1 + \\
&+ \int_{\partial S_{i\sigma}} [\Phi(\mathbf{r}) (\sigma(\mathbf{r}) \nabla \Phi(\mathbf{r}))] \cdot \vec{d}s_1 = \frac{1}{2} \int_{\partial S_{12}} [(\sigma(\mathbf{r}) \nabla \Phi^2(\mathbf{r}))] \cdot \vec{d}s_1 \\
&\quad + \Omega \nabla \cdot \langle [\Phi(\mathbf{r}) (\sigma(\mathbf{r}) \nabla \Phi(\mathbf{r}))] \rangle_f = \\
&= \frac{1}{2} \int_{\partial S_{12}} [(\sigma(\mathbf{r}) \nabla \Phi^2(\mathbf{r}))] \cdot \vec{d}s_1 + \frac{\Omega}{2} \nabla \cdot \langle [(\sigma(\mathbf{r}) \nabla \Phi^2(\mathbf{r}))] \rangle_f. \quad (59)
\end{aligned}$$

where  $\partial S_f = \partial S_{12} + \partial S_{i\sigma}$  is the whole surface bounding the fluid volume  $\Omega_f$ , and  $\partial S_{i\sigma}$  is the surface of input and output of  $\Omega_f$  in the  $\Omega$ .

As one can see in the development of (58) the averaging theorem is being violated. This is a major departure from heterogeneous spacial differentiation rules established through the VAT theorems. Note, that the averaging here is performed over the entire volume of the stated problem  $\Omega$ ,  $\Omega_f$  meaning that the REV is equal to  $\Omega$ . The general forms of the equations for effective coefficients are not done here as was done for temperature fields as the same questions about many versions, applicability of current methods and variance in interpretation are present.



Similar averaged equations result for the 2nd phase. The presence of interface integrals and other terms make experimental data reduction models for heterogeneous medium very different from established methods for homogeneous fields. If the data from heterogeneous media experiments are to become meaningful, the interface phenomena will, themselves, need to be measured in some fashion.

VAT based models were recently developed while modeling electrodynamic properties of a liquid-impregnated porous ferrite media (Ponomarenko et al., 1999), coupled electrostatic-diffusion processes in composites Travkin et al., (1998) and to analyze heat conductivity experimental data in high  $T_c$  superconductors (Travkin and Catton, 1998b). Powders of ferrites with NFM frequency in the microwave range were used as the porous magnetic media in (Ponomarenko et al., 1999). The search for tunable levels of reflection and absorption of electromagnetic waves was conducted using an arbitrarily chosen morphologies. The need for more close cooperation between experimentalists and modelers became obvious.

## 7. CONCLUSIONS

Efforts to advance the theoretical developments and modeling heterogeneous media based on VAT descriptions into experimental practice is in its initial stages and actually does not exist as a field of study. The goal of this effort was to outline a few of the more frequently addressed issues arising from experimental studies of effective conductivities (thermal and electrical) and effective average exchange coefficients in heterogeneous media.

The principal electrophysical properties of porous media are reviewed and experimental data is classified. Different methods of calculating complex permittivity of porous media are compared: composite approximation, Bergman-Milton theory, Grain Consolidation Model, local porosity theory, etc.

The development of procedures for determining the effective bulk electrical potential field coefficient description are explained in terms of VAT. Many VAT models of thermal, momentum and mass transport in heterogeneous media have features like those developed above and have been found to be treatable with closure methods reported elsewhere. The application of VAT models to the issues of effective coefficients in heterogeneous media lead to multivariant problems. Unlike in homogeneous medium, there are few coefficients that can be derived for heterogeneous medium transport coefficients that have effective characteristics.

The challenge at present is for experimentalists to begin to realize the difference in data obtained when a medium is heterogeneous rather than homogeneous so that the correct measurements will be made. Methods and procedures for reducing experimental data will need to be developed. A basic methodology for electrodynamic experimental data reduction for porous and heterogeneous media is suggested in this work.

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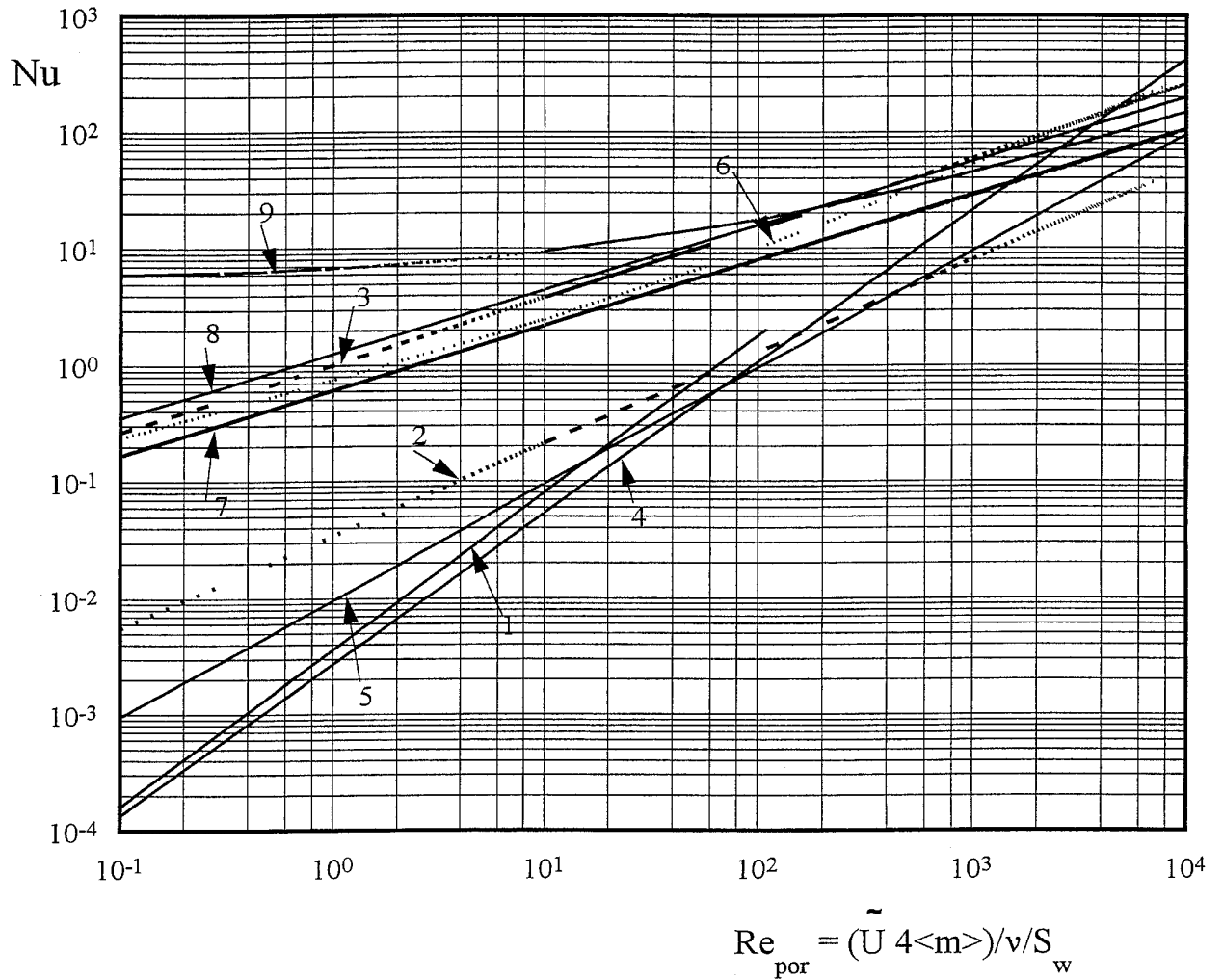


Fig. 1 Internal effective heat transfer coefficient in porous media, reduced based on VAT scale transformations in experiments by: 1) Kar and Dybbs (1982) for laminar regime; 2) Rajkumar (1983); 3) Achenbach (1995); 4) Younis and Viskanta (1993); 5) Galitseysky and Moshayev (1993); 6) Kokorev et al. (1987); 7) Gortyshov (1987); 8) Kays and London (1984); 9) Heat Exchangers Design Handbook (1983)