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STATISTICS OF MATHEMATICAL TWO-SCALE CLOSURE OF MOMENTUM, HEAT AND CHARGE TRANSPORT PROBLEMS WITH STOCHASTIC ORIENTATION OF POROUS MEDIUM CAPILLARIES

V. S. Travkin, K. Hu, I. Catton
Department of Mechanical and Aerospace Engineering
University of California, Los Angeles
Los Angeles, CA 90095

ABSTRACT

The history of stochastic capillary porous media transport problem treatments almost corresponds to the history of porous media transport developments. Volume Averaging Theory (VAT), shown to be an effective and rigorous approach for study of transport (laminar and turbulent) phenomena, is used to model flow and heat transfer in capillary porous media. VAT based modeling of pore level transport in stochastic capillaries results in two sets of scale governing equations. This work shows how the two scale equations could be solved and how the results could be presented using statistical analysis. We demonstrate that stochastic orientation and diameter of the pores are incorporated in the upper scale simulation procedures. We are treating this problem with conditions of Bi for each pore is in a range when $Bi \lesssim 0.1$ which allows even greater distinction in assessing an each additional differential, integral, or integral-differential term in the VAT equations.

Nomenclature

Bi	-	$= \frac{h \cdot d_h}{k_s}$, Biot Number $[-]$
c_p	-	specific heat $[J/(kg \cdot K)]$
d_h	-	hydraulic dynameter, $[m]$
ds	-	interphase differential area in porous medium $[m^2]$
∂S_w	-	internal surface in the REV $[m^2]$
f	-	friction factor
$\widetilde{f}$	-	averaged over $\Delta\Omega_f$ value f
$< f >_f$	-	value f, averaged over $\Delta\Omega_f$ in a REV
$\widehat{f}$	-	value f morpho-fluctuation in a Ω_f
h	-	heat transfer coefficient $[W/(m^2K)]$
k	-	thermal conductivity $[W/(mK)]$
ν_T	-	averaged turbulent eddy viscosity $[m^2/s]$
k_c	-	turbulent diffusion coefficient $[m^2/s]$
k_T	-	turbulent eddy thermal conductivity $[W/(mK)]$

$\langle m \rangle$ - averaged porosity [-]
Pr - Prandtl number [−]
p - pressure $[Pa]$
q - heat flux $[W/m^2]$
R - pore diameter [m]
Re_{por} - Reynolds number of pore hydraulic diameter [-]
S_w - specific surface of a porous medium $\partial S_w/\Delta\Omega$ $[1/m]$
T - temperature $[K]$
u, w - velocity in x,z-direction $[m/s]$

Subscripts

f - fluid phase
i - component of turbulent vector variable
cr - cross section
s - solid phase
T - turbulent
w - wall

Superscripts

$\sim$ - value in fluid phase averaged over REV
$-$ - mean turbulent quantity

Greek letters

$\Delta\Omega$ - representative elementary volume (REV) $[m^3]$
$\Delta\Omega_f$ - pore volume in a REV $[m_3]$
$\Delta\Omega_s$ - solid phase volume in a REV $[m_3]$
ν - kinematic viscosity $[m^2/s]$
ϱ - density $[kg/m^3]$
μ - dynamic viscosity $[kg/ms]$

Introduction

As that mentioned by Kaviany (1995), among the fundamental studies for transport phenomena over rough surfaces in porous media, pore-level transport attracts more attention due to the need for a better understanding and prediction of fluid flow and heat and mass transfer. Pore level models of porous medium include morphological, or probabilistic parameters that are unique to this length scale. One popular pore level model to study the porous media is the network model. Capillary-bundle models are the simplest type network. They incorporate flow variation, but in neglecting the effects of inter-connectivity and tortuosity they are useful primarily as conceptual tools. Tortuosity is neglected because each pore is straight.

The parameters that dictate a network's geometry are its spatial dimension (i.e., 2-D or 3-D), grid pattern (which maybe regular or irregular), bond-size distribution, and coordination number (the value and whether it is constant). Numerous studies have been carried out with increasingly sophisticated rules to describe capillary equilibrium and simultaneous flow of fluids, such as Goode and Ramakrishnan (1993), Thompson and Fogler (1997), Rieckmann and Keil (1997) and Thauvin and Mohanty (1998).

In a network morphology, since each of the significant morphology elements can be random, researchers have gradually developed a more sophisticated description to treat random network morphologies with up to 5 degrees of randomness: (1) pore surface roughness, (2) pore diameter, (3) pore length, (4) pore pathway between nodes, and (5) pore cross-sectional shape. Travkin and Catton (1999) proved that statistical analysis is an effective way to study the behavior porous media capillary network.

Volume averaging is a widely used technique in which a macroscopic momentum equation is derived from Navier-Stokes equations and turbulent regime equations averaged over a small representative elementary volume. During the averaging process, hydrodynamic information from the pore scale is retained, but it is reintroduced in the form of unknown fluctuations that can be determined experimentally or derived for simple pore structures.

In this paper, we continue to study the capillary models introduced in works by Gratton et. al. (1994), Travkin and Catton (1999) and Hu et al. (2001). The capillary morphology shown in Figure 1 is used as a morphology model for volume averaged capillary network model development. Travkin and Catton (1999) demonstrated a two scale solution for volume averaging theory (VAT) model of momentum transport in a sim-

ple case of straight capillaries medium. For heat transfer in a network morphology, the temperature field is not homogeneous. Two scale solution for energy transport is addressed in this work. Statistical method is used to analyze the capillary two scale solution.

The problem of lower scale heat transport in straight capillary is a conjugate problem. The analytical methods of solution of conjugated problems presented by Luikov et al. (1970) and Luikov (1974) are used in this paper for solving lower scale conjugated convective heat transfer problems. Previous studies have shown that VAT models are effective for the study of upper scale heat transport in straight capillary morphology, see Travkin and Catton (1998), Zanotti and Carbonell (1984) and Yuan et al. (1991). But the approach used in this paper to formulate closure and to find solution if current VAT problem is essentially different to those previous work.

In this paper a usual homogeneous momentum transport and heat transport in each pore and its surrounding solid phase are defined as lower scale transport problems. The upper scale physical and mathematical model and governing equations are the VAT (heterogeneous) equations described in following sections.

Momentum Development for Flow in Porous Media by Using Network Model for Closure of VAT Statement

In order to show how equations and consistent closure models based on VAT are developed for transport of momentum in porous media using a network model, a one dimensional straight pore morphology shown in Figure 1 is chosen. For this kind of capillary morphology model, theoretical solutions for velocity profile and dispersion coefficients can be obtained.

Upper Scale Momentum Equation

The VAT based 1-D momentum equation for turbulent flow in porous medium (Travkin and Catton, 1995)

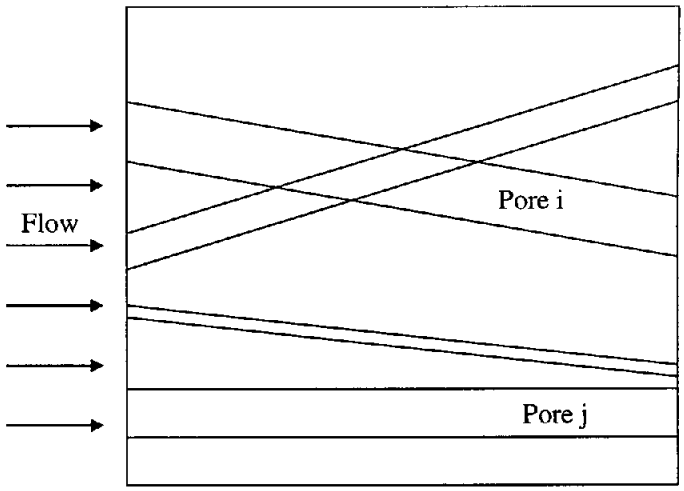

Figure 1: Straight capillary morphology model

shown in Figure 1 is

$$\frac{\partial}{\partial x}\left(\langle m\rangle\left(\widetilde{\nu}_T+\nu\right)\frac{\partial\widetilde{\overline{u}}}{\partial x}\right) \tag{1}$$
$$+\frac{\partial}{\partial x}\left(\left\langle\widetilde{\nu}_T\frac{\partial\,\widehat{\overline{u}}}{\partial x}\right\rangle_f\right)+\frac{\partial}{\partial x}\left(\left\langle-\widehat{\overline{u}}\,\widehat{\overline{u}}\right\rangle_f\right)$$
$$=\langle m\rangle\widetilde{\overline{u}}\frac{\partial\,\widetilde{\overline{u}}}{\partial x_j}+\frac{1}{\varrho_f\Delta\Omega}\int_{\partial S_w}\overline{p}\vec{ds}$$
$$+\frac{1}{\varrho_f}\frac{\partial}{\partial x}\left(\langle m\rangle\widetilde{\overline{p}}\right)-\frac{1}{\Delta\Omega}\int_{\partial S_w}\left(\nu_T+\nu\right)\frac{\partial\overline{u}}{\partial x_i}\cdot\vec{ds}$$

Derivation and basics of VAT for turbulent transport is given in Primak et al. (1986), Scherban et al. (1986) and Travkin and Catton (1995). One can embrace the all three possible flow regimes, laminar, intermediate and turbulent, in each of the pores. For this momentum equation closure is obtained by modeling the overall drag resistance factor, c_d, for the second and third terms on the right hand side, and the second and third morpho-diffusive terms on the left-hand side. We are assuming fully developed momentum regime, which means that in the adopted medium eliminated the differential terms in the above momentum equation, so the resulting VAT based momentum equation

becomes

$$\frac{1}{\varrho_f}\frac{\partial}{\partial x}\left(\langle m\rangle\widetilde{\overline{p}}\right) \tag{2}$$
$$= -\frac{1}{\varrho_f \Delta\Omega}\int_{\partial S_w}\overline{p}\vec{ds} + \frac{1}{\Delta\Omega}\int_{\partial S_w}\left(\nu_T+\nu\right)\frac{\partial\overline{U}}{\partial x_i}\cdot\vec{ds}$$

For closure of the integral terms in the above equations, one has to integrate over the interface surface (between solid and fluid) in the REV.

Lower Scale Momentum Transport in Pores

The relationship between the lower scale velocity and upper scale velocity and the relationship between average velocity and fluctuation velocity are shown in Travkin and Catton (1999) and Hu et. al (2001). The momentum equation turbulent and intermediate regime treatment is reported in Travkin et al. (2001). This development is possible only because the momentum transport in each pore is independent from other pores.

Heat Transfer Analysis for Network Model

For heat transfer in the medium shown in Figure 1, the temperature field is not homogeneous. We would like to use the two temperature model developed in Travkin and Catton (1995) as upper scale governing equation.

Upper Scale Governing Equations

Travkin et al. (1993) and Travkin and Catton (1995) showed that the proper form for heat transfer equation in the fluid phase with primarily 1-D turbulent convective heat transfer is

$$c_{pf}\varrho_f\langle m\rangle\widetilde{\overline{u}}\frac{\partial\widetilde{\overline{T}}_f}{\partial x} \tag{3}$$
$$= \frac{\partial}{\partial x}\left[\langle m\rangle\left(\widetilde{k}_T+k_f\right)\frac{\partial\widetilde{\overline{T}}_f}{\partial x}\right]$$
$$+\frac{\partial}{\partial x}\left(\left\langle\widehat{k}_T\frac{\partial\widehat{\overline{T}}_f}{\partial x}\right\rangle_f\right)$$
$$+c_{pf}\varrho_f\frac{\partial}{\partial x}\left[\langle m\rangle\left\{-\widehat{\overline{T}}_f\,\widehat{\overline{u}}\right\}_f\right]$$
$$+\frac{\partial}{\partial x}\left[\frac{\left(\widetilde{k}_T+k_f\right)}{\Delta\Omega}\int_{\partial S_w}\widehat{\overline{T}}_f\vec{ds}\right]$$
$$+\frac{1}{\Delta\Omega}\int_{\partial S_w}\left(k_T+k_f\right)\frac{\partial\overline{T}_f}{\partial x_i}\cdot\vec{ds},$$

while in the neighboring solid phase, using constant conductivity, the corresponding equation is

$$\frac{\partial}{\partial x}\left[\left(1-\langle m\rangle\right)k_s\frac{\partial\left\{T_s\right\}_s}{\partial x}\right] \tag{4}$$
$$= -\frac{\partial}{\partial x}\left[\frac{k_s}{\Delta\Omega}\int_{\partial S_w}\widehat{T}_s\vec{ds}_1\right]$$
$$-\frac{1}{\Delta\Omega}\int_{\partial S_w}k_s\frac{\partial T_s}{\partial x_i}\cdot\vec{ds}_1$$

where

$$\vec{ds}_1 = -\vec{ds}$$

For laminar flow, the two-temperature energy equation is simplified to

$$c_{pf}\varrho_f\langle m\rangle\widetilde{u}\frac{\partial\widetilde{T}_f}{\partial x} \tag{5}$$
$$= \frac{\partial}{\partial x}\left[\langle m\rangle k_f\frac{\partial\widetilde{T}_f}{\partial x}\right]$$
$$+c_{pf}\varrho_f\frac{\partial}{\partial x}\left[\langle m\rangle\left\{-\widehat{T}_f\,\widehat{u}\right\}_f\right]$$
$$+\frac{\partial}{\partial x}\left[\frac{k_f}{\Delta\Omega}\int_{\partial S_w}\widehat{T}_f\vec{ds}\right]$$
$$+\frac{1}{\Delta\Omega}\int_{\partial S_w}k_f\frac{\partial T_f}{\partial x_i}\cdot\vec{ds},$$

Lower Scale Governing Equations

At the lower scale, the flow in each pore is homogeneous flow. So the governing equations can be much more simplified. One feature is different, it is the dimensions of space - 2D statements. For convenience, equations are written in cylindrical coordinate system instead of Cartesian coordinate system.

Governing Equation in the Solid Phase In the solid phase, the steady state heat conduction equation in pore i can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_{s_i}}{\partial r}\right)+\frac{\partial}{\partial x}\left(\frac{\partial T_{s_i}}{\partial x}\right)=0 \tag{6}$$

The boundary conditions for each pore are

$$r=R_i, \quad T_{s_i}=T_{s,win_i} \tag{7}$$

$$r=R_i+\delta_i, \quad T_{s_i}=T_{s,wout_i} \tag{8}$$

where δ_i means the thickness of the cylindrical layer around each pore where the temperature drops from T_{s,win_i} to $T_{s,wout_i}$. $T_{s,wout_i}$ is a constant temperature outside the solid cylindrical layer. Here negligible variations of heat flux at the external surface $r=R_i+\delta_i$ is assumed.

Fluid Phase Governing Equations in the Pore Laminar regime treatment has been studied in Hu et. al (2001). In a cylindrical system, the lower scale energy equation for turbulent regime heat transfer in each pore can be written as

$$\begin{aligned}&\overline{u}_i(r)\ r\frac{\partial \overline{T}_{f_i}(z_l,r)}{\partial x} \\ =\ &\frac{\partial}{\partial r}\left[r\left(\frac{k_f}{c_{pf}\,\varrho_f}+\frac{\nu_T}{\mathrm{Pr}_T}\right)\frac{\partial \overline{T}_{f_i}(z_l,r)}{\partial r}\right]\end{aligned} \tag{9}$$

Symmetrical boundary condition can be applied to equation (9) at the center line of the straight pore

$$r=0, \quad \frac{\partial \overline{T}_{f_i}(z_l,r)}{\partial r}=0. \tag{10}$$

The conjugate boundary conditions are

$$\overline{T}_{f_i}|_{r=R_i}=T_{s_i}|_{r=R_i}=T_{s,win_i}(z_l) \tag{11}$$

and

$$\vec{q}_{s_i}|_{r=R_i}=\vec{q}_{f_i}|_{r=R_i}. \tag{12}$$

Relationships between lower scale temperature and upper scale temperature To find out the average over the fluid phase within the REV one needs to use the lower scale results.

$$\begin{aligned}\langle \overline{T}_f\rangle_f &= \frac{1}{\Delta\Omega}\int_{\Delta\Omega_f}\overline{T}_f d\omega \\ &= \langle m\rangle\left\{\overline{T}\right\}_f=\langle m\rangle\,\widetilde{\overline{T}},\end{aligned} \tag{13}$$

and the average over the solid volume within the REV is

$$\begin{aligned}\langle T_s\rangle_s &= \frac{1}{\Delta\Omega}\int_{\Delta\Omega_s}T_s d\omega \\ &= (1-\langle m\rangle)\left\{T_s\right\}_s=(1-\langle m\rangle)\,\widetilde{T}_s\end{aligned} \tag{14}$$

and

$$\widehat{\overline{T}}=T-\widetilde{\overline{T}}$$

The details of lower scale simulation could be found in Hu et al. (2001).

Closure of Morphodependent additional VAT terms in the upper scale heterogeneous equations

For closure of the integral terms

$$\frac{1}{\varrho_f\Delta\Omega}\int_{\partial S_w}p\vec{ds} \tag{15}$$

and

$$\frac{\nu}{\Delta\Omega}\int_{\partial S_w}\frac{\partial U}{\partial x_i}\cdot\vec{ds} \tag{16}$$

in the momentum equations, one has to integrate over the interface surface ∂S_w (between solid and fluid) in the REV.

While to solve the energy equations (3) and (4), the closure of the following terms in upper scale VAT equations need to be found out. They are unknown terms in fluid phase heat transport equation

$$\frac{\partial}{\partial x}\left[\langle m\rangle\left\{-\widehat{\overline{T}}_f\,\widehat{\overline{u}}\right\}_f\right] \tag{17}$$

$$\frac{\partial}{\partial x}\left[\frac{k_f}{\Delta\Omega}\int_{\partial S_w}\widehat{\overline{T}}_f\vec{ds}\right] \tag{18}$$

$$\frac{1}{\Delta\Omega}\int_{\partial S_w}k_f\frac{\partial \overline{T}_f}{\partial x_i}\cdot\vec{ds} \tag{19}$$

and unknown terms in solid phase heat transport equation

$$\frac{\partial}{\partial x}\left[\frac{\{k_s\}_s}{\Delta\Omega}\int_{\partial S_w}\widehat{T}_s\vec{ds}_1\right] \tag{20}$$

$$\frac{1}{\Delta\Omega}\int_{\partial S_w}k_s\frac{\partial T_s}{\partial x_i}\cdot\vec{ds}_1 \tag{21}$$

Turbulent and transitional equations need one more term for closure temperature equation.

$$\frac{\partial}{\partial x}\left(\left\langle\widehat{k}_T\frac{\partial\widehat{\overline{T}}_f}{\partial x}\right\rangle_f\right)$$

And take nonlinear conductivity coefficient in the integral terms.

Mathematical Expressions for Calculation of Each Term in the Network Upper Scale Equations

The upper scale equations are written in Cartesian coordinate. But it is easier to calculate the lower scale local fields in the local Cylindrical system. To make things simpler we start with the straight parallel pores. Figure 2 shows the relationship between cylindrical coordinate and Cartesian coordinate. The relationships between Cartesian coordinate and cylindrical coordinate when z_l axis and x axis coincides ($\alpha_i = 0$) are

$$\begin{aligned} x &= z_l \\ y &= r\cos\varphi \\ z &= r\sin\varphi \end{aligned} \tag{22}$$

The unit specific surface area ds could be calculated through the following steps

$$\begin{aligned} E &= \left(\frac{\partial x}{\partial\varphi}\right)^2+\left(\frac{\partial y}{\partial\varphi}\right)^2+\left(\frac{\partial z}{\partial\varphi}\right)^2 \\ &= R_i^2\sin^2\varphi+R_i^2\cos^2\varphi \\ F &= \frac{\partial x}{\partial\varphi}\frac{\partial x}{\partial z_l}+\frac{\partial y}{\partial\varphi}\frac{\partial y}{\partial z_l}+\frac{\partial z}{\partial\varphi}\frac{\partial z}{\partial z_l}=0 \\ G &= \left(\frac{\partial x}{\partial z_l}\right)^2+\left(\frac{\partial y}{\partial z_l}\right)^2+\left(\frac{\partial z}{\partial z_l}\right)^2=1 \end{aligned} \tag{23}$$

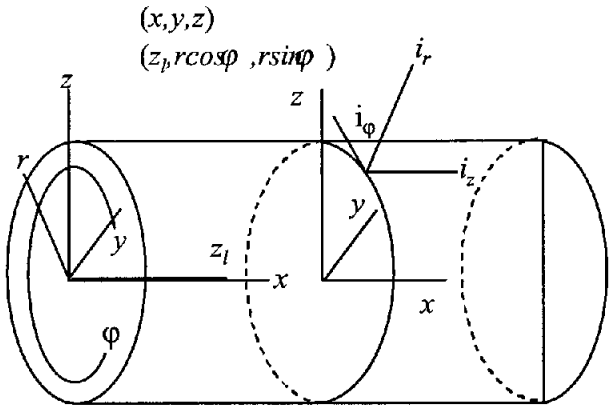

Figure 2: Transformation between upper scale Cartesian coordinate system and pore local cylindrical system.

So

$$\begin{aligned} ds &= \sqrt{EG-F^2}dz_l d\varphi \\ &= R_i dz_l d\varphi, \end{aligned} \tag{24}$$

As we already discussed, to make the upper scale equations solvable, we need to calculate those unknown terms through the results of lower scale equations.

In cylindrical coordinates, the operation factor has the transformation relationship

$$\begin{aligned} & (\nabla\mathbf{V})_{ij}\,\mathbf{n}_i \\ = \; & \mathbf{n}_i\cdot(\nabla\mathbf{V})_{ij} \\ = \; & \left(n_r\frac{\partial V_r}{\partial r}+n_\varphi\frac{1}{r}\frac{\partial V_r}{\partial\varphi}+n_{z_l}\frac{\partial V_r}{\partial z_l}\right)\mathbf{i_r} \\ & +\left(n_r\frac{\partial V_\varphi}{\partial r}+n_\varphi\frac{1}{r}\frac{\partial V_\varphi}{\partial\varphi}+n_{z_l}\frac{\partial V_\varphi}{\partial z_l}\right)\mathbf{i}_\varphi \\ & +\left(n_r\frac{\partial V_{z_l}}{\partial r}+n_\varphi\frac{1}{r}\frac{\partial V_{z_l}}{\partial\varphi}+n_{z_l}\frac{\partial V_{z_l}}{\partial z_l}\right)\mathbf{i_z}, \end{aligned} \tag{25}$$

Calculation of upper scale laminar unknown terms In the upper scale equations, the terms for laminar regime VAT equations need to be calculated are two terms from VAT momentum equation, three terms from VAT fluid phase heat transport equation and two terms from VAT solid phase heat transport equation. For straight

pore capillary network, given lower scale results those unknown terms (16), (17), (18), (19), (20) and (21) could easily be solved.

For parallel flow in straight pores we have the following relations

$$\begin{aligned} U_{z_l}(r) &\neq 0 \\ U_r &= 0 \\ U_\theta &= 0 \end{aligned} \tag{26}$$

and in Cartesian coordinate

$$\begin{aligned} U_x(x,y,z) &= U_{z_l}(r) \neq 0 \\ U_y(x,y,z) &= 0 \\ U_z(x,y,z) &= 0 \end{aligned} \tag{27}$$

It's obvious that unknown term (15) in laminar regime for straight pore morphology becomes zero because $\vec{n} = (1,0,0)|_{\partial S_w}$. For example in pore #1,

$$\begin{aligned} & -\frac{1}{\varrho_f \Delta\Omega} \int_{\partial S_w} p(z_l)\, \vec{ds} \\ = & -\frac{1}{\varrho_f \Delta\Omega} \int_{\partial S_w} \vec{n}_1 p(z_l)\, ds \\ = & -\frac{1}{\varrho_f \Delta\Omega} \int_{\partial S_w} p(z_l)\,(n_x\mathbf{i} + n_y\mathbf{j} + n_z\mathbf{k})\, ds \end{aligned} \tag{28}$$

in the local cylindrical system coordinates for any of straight pore when axis $\vec{i}_z = \mathbf{i}$, then $\vec{n} = (\mathbf{i}_r, 0, 0)|_{\partial S_w}$, so

$$\begin{aligned} & -\frac{1}{\varrho_f \Delta\Omega} \int_{\partial S_w} p(z_l)\,(n_x\mathbf{i} + n_y\mathbf{j} + n_z\mathbf{k})\, ds \\ = & -\frac{1}{\varrho_f \Delta\Omega} \int_{\partial S_w} p(z_L)\,(1,0,0)|_{\partial S_w}\, ds \\ = & \left(-\frac{1}{\varrho_f \Delta\Omega} \int_{\partial S_w} p ds\right) \mathbf{i}_r + 0\mathbf{i}_\varphi + 0\mathbf{i}_z \\ = & -\frac{1}{\varrho_f \Delta\Omega} \int_{\partial S_w} p(z_l)\, R_1 dz_l d\varphi \mathbf{i}_r + 0\mathbf{i}_\varphi + 0\mathbf{i}_z \\ = & \ 0 \end{aligned} \tag{29}$$

For term (16) in laminar regime momentum equation

$$\frac{\nu}{\Delta\Omega} \int_{\partial S_w} \frac{\partial U}{\partial x_i} \cdot \vec{ds} = \frac{\nu}{\Delta\Omega} \int_{\partial S_w} \vec{n}_i \cdot \frac{\partial U_j}{\partial x_i} ds. \tag{30}$$

In Cartesian coordinate, this term is expressed as

$$\begin{aligned} & \frac{\nu}{\Delta\Omega} \int_{\partial S_w} \vec{n}_i \cdot \frac{\partial U_j}{\partial x_i} ds \\ = & \frac{\nu}{\Delta\Omega} \int_{\partial S_w} \left(n_x \frac{\partial U_x}{\partial x} + n_y \frac{\partial U_x}{\partial y} + n_z \frac{\partial U_x}{\partial z}\right) ds\, \vec{i} \\ & + \frac{\nu}{\Delta\Omega} \int_{\partial S_w} \left(n_x \frac{\partial U_y}{\partial x} + n_y \frac{\partial U_y}{\partial y} + n_z \frac{\partial U_y}{\partial z}\right) ds\, \vec{j} \\ & + \frac{\nu}{\Delta\Omega} \int_{\partial S_w} \left(n_x \frac{\partial U_z}{\partial x} + n_y \frac{\partial U_z}{\partial y} + n_z \frac{\partial U_z}{\partial z}\right) ds\, \vec{k} \end{aligned} \tag{31}$$

While in cylindrical system coordinates,

$$\begin{aligned} \vec{ds} &= \vec{n} ds \\ \vec{n} &= (\mathbf{i}_r, 0, 0)|_{\partial S_w} \end{aligned} \tag{32}$$

Then

$$\begin{aligned} & \frac{\nu}{\Delta\Omega} \int_{\partial S_w} \vec{n}_i \cdot \frac{\partial U_j}{\partial x_i} ds \\ = & \frac{\nu}{\Delta\Omega} \int_{\partial S_w} \left(n_r \frac{\partial U_{z_l}}{\partial r}\right) ds\, \vec{i}_z \\ = & \sum_{k=1}^{N} \frac{2\pi R_k \nu}{\Delta\Omega} \int_{x_i}^{x_{i+1}} \frac{\partial U_{k,z_l}(r)}{\partial r} dz_l \end{aligned} \tag{33}$$

and in each pore k

$$\begin{aligned} & \frac{2\pi R_k \nu}{\Delta\Omega} \int_{x_i}^{x_{i+1}} \frac{\partial U_{z_l}(r)}{\partial r} dz_l \\ = & \frac{2\pi R_k \nu}{\Delta\Omega} \left.\frac{\partial U_{k,z_l}(r)}{\partial r}\right|_{r=R_k} (x_{i+1} - x_i). \end{aligned} \tag{34}$$

It could be derived that velocity in pore k is

$$\begin{aligned} & U_{k,z_l}(r) \\ = & \frac{1}{4\mu}\left(-\frac{dp}{dx}\right)(R_k^2 - r^2) \end{aligned} \tag{35}$$

This above expression for known velocity profile in each of straight pores yields an analytical formula for term (16) in each REV i

$$\begin{aligned} & \frac{\nu}{\Delta\Omega}\int_{\partial S_w}\frac{\partial U}{\partial x_i}\cdot\vec{ds} \\ = & \sum_{k=1}^{N}\frac{2\pi R_k\nu}{\Delta\Omega}\left.\frac{\partial U_{k,z_l}(r)}{\partial r}\right|_{r=R_k}(x_{i+1}-x_i) \\ = & \sum_{k=1}^{N}\frac{2\pi R_k\nu}{\Delta\Omega}\left(\frac{1}{2\mu}\frac{dp}{dx}R_k\right)(x_{i+1}-x_i) \\ = & \sum_{k=1}^{N}\frac{\pi\nu}{\Delta\Omega}\left(\frac{1}{\mu}\frac{dp}{dx}\right)(x_{i+1}-x_i) \end{aligned} \tag{36}$$

For term (17) in laminar fluid heat transport equation, it is a differential term. So it can be easily solved through grid equations. That is

$$\begin{aligned} & \frac{\partial}{\partial x}\left(\langle m\rangle\left\{-\widehat{T}_{fk}\widehat{u}_k\right\}_f\right) \\ = & \frac{\left(\langle m\rangle\left\{-\widehat{T}_{fk}\widehat{u}_k\right\}_f\right)_{i+1}-\left(\langle m\rangle\left\{-\widehat{T}_{fk}\widehat{u}_k\right\}_f\right)_i}{\Delta x} \end{aligned} \tag{37}$$

where $\widehat{T}_f(r,x=z_l) = \left(T_f(r,z_l)-\{T(x)\}_f\right)$ is the volume fluctuation temperature, and $\widehat{u}_i = \widehat{u}_i(r) = \left(U_i(r)-\{U\}_f\right)$ is the volume fluctuation velocity. In each REV, $T_f(r,x)$ is the number result of lower scale equations, and $\{T\}_f$ is the volume averaged velocity in REV. So $\widehat{T}_f$ could be easily calculated in each REV.

For unknown term (18), it could be calculated from the following correlations

$$\begin{aligned} & \frac{\partial}{\partial x}\left(\frac{k_f}{\Delta\Omega}\int_{\partial S_w}T_f\vec{ds}\right) \\ = & \frac{\partial}{\partial x}\left(\sum_{k=1}^{N}\left(2\pi R_k\frac{k_f}{\Delta\Omega}\int_{x_i}^{x_{i+1}}T_{fk}dz_l\right)\right) \end{aligned} \tag{38}$$

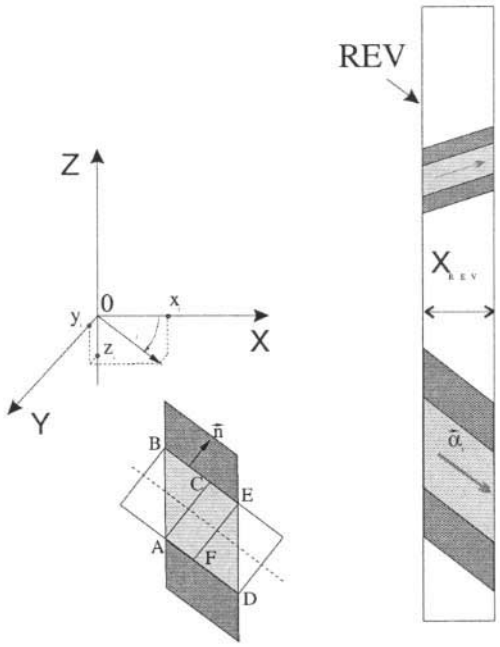

Figure 3: One pore VAT calculation

For term (19) in pore k,

$$\begin{aligned} & \frac{1}{\Delta\Omega}\int_{\partial S_w}k_f\frac{\partial T_{fk}}{\partial x_i}\cdot\vec{ds} \\ = & \frac{2\pi R_k k_f}{\Delta\Omega}\int_{x_i}^{x_{i+1}}\frac{\partial T_{fk}}{\partial r}dz_L. \end{aligned} \tag{39}$$

In the solid phase energy transport equation the terms (20) and (21) are identical to terms (18) and (19) respectively from the fluid phase energy transport equation. Because the temperatures and heat fluxes on the interface surface are equal

$$T_f|_{\partial S_w} = T_s|_{\partial S_w}.$$

Coordinates Transformation

To start the upper scale calculation, we start with assigning volume for pores randomly angled in the REV (see Figure 3) in spherical system of coordinates where φ_s and θ_s are randomly assigned angle values in ranges

$$\begin{aligned} -\frac{\pi}{2} < & \varphi_s < \frac{\pi}{2} \\ 0 < & \theta_s < \pi \end{aligned} \tag{40}$$

also the thickness of REV is

$$0 \leqslant x < x_{REV} \tag{41}$$

and Cartesian initial coordinates are

$$\begin{aligned} x &= r_s \sin\theta_s \cos\varphi \\ y &= r_s \sin\theta_s \sin\varphi \\ z &= r_s \cos\theta_s \end{aligned} \tag{42}$$

where $\overrightarrow{r_s}$ is the vector (module r_s) along any selected pore.

Assigning the plane crossing the vectors $\left(\overrightarrow{k}, \overrightarrow{r_s} = \overrightarrow{r_i}\right)$ is

$$\begin{aligned} \overrightarrow{r_s} &= \overrightarrow{r_i} \\ &= \cos(r_s, x)\ \overrightarrow{i} + \cos(r_s, y)\ \overrightarrow{j} + \cos(r_s, z)\ \overrightarrow{k} \end{aligned} \tag{43}$$

where

$$\begin{aligned} \cos(r_s, x) &= \sin\theta_s \cos\varphi \\ \cos(r_s, y) &= \sin\theta_s \sin\varphi \\ \cos(r_s, z) &= \cos\theta_s \end{aligned} \tag{44}$$

So

$$\overrightarrow{k} = (1, 0, 0) \tag{45}$$

is like assigning the plane

$$(r - r_1)(r - r_2)(r - r_3) = 0$$

or

$$A_p x + B_p y + C_p z = 0 \tag{46}$$

where A_p, B_p and C_p are the plane normal's coordinates $\overrightarrow{n}_{(z,r_s)}$ which can be easily calculated based on φ_s and θ_s.

Then we make transformation of our initial Cartesian system coordinate $\sum = \left\{0, \overrightarrow{i}, \overrightarrow{j}, \overrightarrow{k}\right\}$ to a system $\sum_1 = \left\{0, \overrightarrow{i_1}, \overrightarrow{j_1}, \overrightarrow{k_1}\right\}$ via the rotation around normal vector (A_p, B_p, C_p) for θ_s angle. Getting transformation of coordinates in this way

$$\begin{aligned} x_1 &= x\left(\cos\theta_s + \alpha^2(1 - \cos\theta_s)\right) \\ &\quad + y(\gamma \sin\theta_s + \alpha\beta(1 - \cos\theta_s)) \\ &\quad + z(-\beta \sin\theta_s + \alpha\gamma(1 - \cos\theta_s)) \end{aligned} \tag{47}$$

$$\begin{aligned} y_1 &= x(-\gamma \sin\theta_s + \alpha\beta(1 - \cos\theta_s)) \\ &\quad + y\left(\cos\theta_s + \beta^2(1 - \cos\theta_s)\right) \\ &\quad + z(\alpha \sin\theta_s + \beta\gamma(1 - \cos\theta_s)) \end{aligned} \tag{48}$$

$$\begin{aligned} z_1 &= x(\beta \sin\theta_s + \alpha\gamma(1 - \cos\theta_s)) \\ &\quad + y(-\alpha \sin\theta_s + \gamma\beta(1 - \cos\theta_s)) \\ &\quad + z\left(\cos\theta_s + \gamma^2(1 - \cos\theta_s)\right) \end{aligned} \tag{49}$$

where

$$\begin{aligned} \alpha &= \cos\angle\left(x, \overrightarrow{n}_{(z,r_s)}\right) \\ \beta &= \cos\angle\left(y, \overrightarrow{n}_{(z,r_s)}\right) \\ \gamma &= \cos\angle\left(z, \overrightarrow{n}_{(z,r_s)}\right) \end{aligned} \tag{50}$$

One can find out the reverse transformation, which we need to use further. The local calculations in cylindrical systems of coordinate $\sum_{p_i} = \left\{0, \overrightarrow{i_{r_i}}, \overrightarrow{j_{\varphi_i}}, \overrightarrow{k_{z_i}}\right\}$ for each i^{th} pore performed as we outlined before then must be transformed back to systems $\sum_1$ and initial Cartesian coordinate system $\sum$. In this way we still reduce the algorithmic complexity while continue to calculate closure mathematics without compromise. The local lower scale volumes and interface surfaces are transformed from straight parallel morphology and comprise themselves as stated pores and rings.

Random Distribution Pore Size Morphology

The porous media network approach used here is to solve the lower scale transport equations first because lower scale equations have much simpler forms and some of them have theoretical solutions. As soon as the lower scale equations are solved, their solutions can be used to find closure for the upper scale equations and to solve upper scale equation based on the relationships between lower scale variables and upper scale variables.

One of the primary goals of lower scale numerical simulation is to obtain momentum and heat transport results that are exact solutions and for which exact averaging procedures could be accomplished for a broad spectrum of flow regimes and pore diameters. Results were generated for a specified range of diameters then

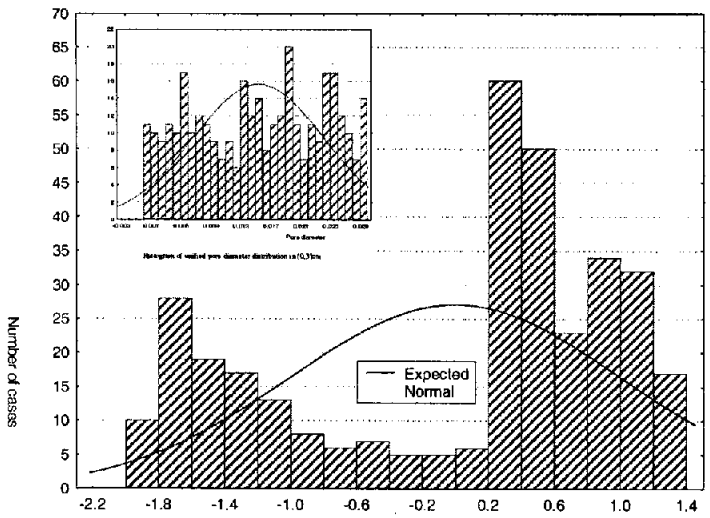

Figure 4: High flow regime velocity histogram for capillary with uniform pore diameter distribution in range (0,3] mm, Repor=4413.9.

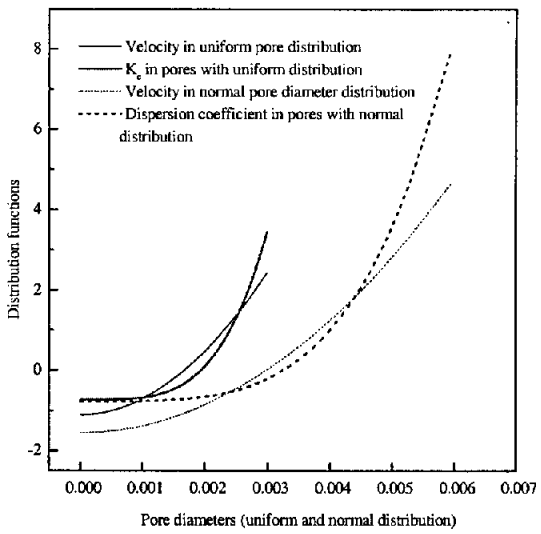

Figure 5: Comparison of laminar regime velocity and dispersion coefficient for straight capillary with normal and uniform pore diameter distribution

complied using an appropriate distribution to guide the sampling. the distributions of pore radii were taken as uniform and normal.

The series of calculations were made for diameters ranging from 0 to 3 mm, including fine pores and pores with diameters on the order of 1mm. They were modeled for different flow regimes in different pores and for various assigned pressure drops. From Figure 4 we can see even for uniform flow regime and uniform pore radius distribution, outcome functions sometimes reflect an extreme shift toward both sides of the results ranges.

Figure 5 shows the difference between uniform and normal straight pore morphology pore diameter distributions medium velocities and dispersion coefficients in laminar regime. It shows that in certain range ($2mm < d_i < 5mm$) these two distributions compare well. There is a big difference for these two distributions at extreme cases.

Figure 6 shows the laminar regime velocities histograms of uniform and normal pore diameter distributions medium. It shows that the velocity histogram in uniformly distributed pores and the velocity histogram in normally distributed pores have nearly same trend when the normalized distribution variables have values > -1.0.

Heat transport is also an important part. Figure 7 depicts interesting tendency of upper scale averaged fluid temperature along flow direction in pores with diameters have a uniform distribution. This figure shows that pore averaged temperature directly correlate to pore diameter given each pore has same inlet fluid temperature. So randomness of capillary pore morphology results in non-linearity of capillary upper scale heat transfer performance.

Summary and Conclusions

Two scale heat transport conjugate problem in capillary porous medium is addressed using a rigorous application of volume averaging theory and statistical method. This two scale approach is carried out in three steps. The first step is to solve equations for lower scale transport in each straight pore. The second step is to use lower scale results in upper scale VAT equations and solve upper scale VAT equation.

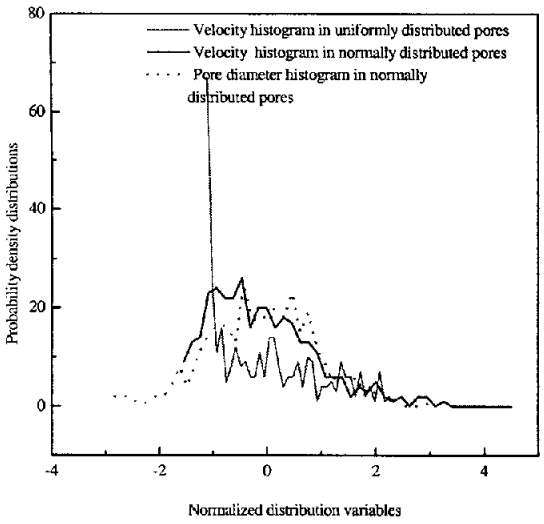

Figure 6: Laminar regime velocity histogram for straight capillary with uniformly and normaly distributed pore diameter

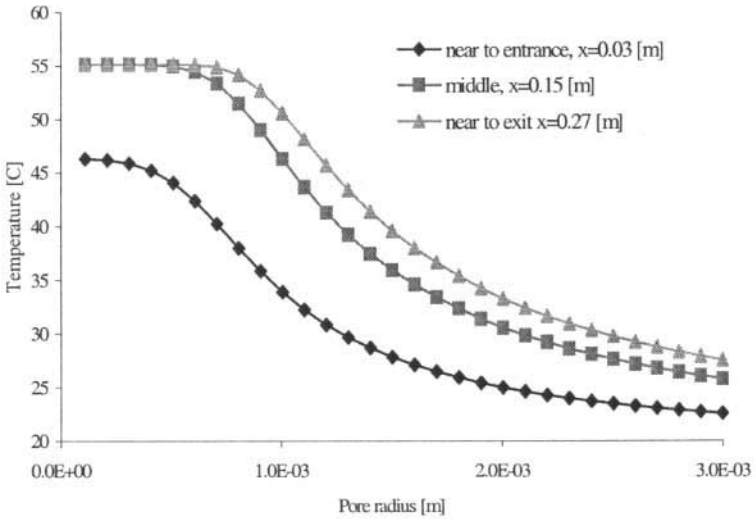

Figure 7: Pore average temperature for different radius pore at different position

Thirdly, the upper scale VAT results are analyzed using statistical method. The problem stated and treated in this paper is part of the problem of capillary network porous medium simulation. Transport in capillary straight pore morphology could be used as a model of membrane transport. The attractive feature of this morphology is that all parameters and characteristics can be evaluated precisely for both scales.

Statistical analysis of capillary network morphology numerical results shows some interesting phenomena. The results also shows that both VAT and statistics are effective ways for the study of pore level capillary network.

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References

Fatt, I., 1956, The network model of porous media: I, II, III, *Trans. AIME*, Vol. 207, pp. 144-163.

Goode, P. A. and Ramakrishnan, T. S., 1993, Momentum transfer across fluid-fluid interfaces in porous media: a network model, *AIChE Journal*, Vol. 39, No. 7, pp. 1124-1134.

Gratton, L., Travkin, V. S. and Catton, I., 1994, Numerical solution of turbulent heat and mass transfer in a stratified geostatistical porous layer for high permeability media, *ASME Paper 94-WA-HT-41.*

Hu, K., Travkin, V. S. and Catton, I., 2001, Two scale hierarchical capillary network model of heat and momentum transport in porous media, *Proceedings of NHTC'01, 35th National Heat Transfer Conference*, June, 10-12, 2001, Anaheim, California.

Kaviany, M. 1995, *Principals of Heat Transfer in Porous Media*, 2nd edition, Springer, New York.

Luikov, A. V., Aleksashenko, V. A. and Aleksashenko, A. A., 1970, Analytical methods of solution of conjugated problems in convective heat transfer, *Int. J. Heat Mass Transfer*, Vol. 14, pp. 1047-1056.

Luikov, A. V., 1974, Conjugate Convective Heat Transfer Problems, *Int. J. Heat Mass Transfer*, Vol. 17, pp. 257-265.

Primak, A. V., Shcherban, A. N. and Travkin, V. S. (1986), Turbulent Transfer in Urban Agglomerations on the Basis of Experimental Statistical Models of Roughness Layer Morphological Properties. In *Transactions World Meteorological Organization Conference on Air Pollution Modeling and its Application*, 2, pp. 259-266, WMO, Geneva.

Rieckmann, C. and Keil, F. J., 1997, Multicomponent diffusion and reaction in three-dimensional networks: general kinetics, *Ind. Eng. Chem. Res.*, Vol. 36, pp. 3275-3281.

Scherban, A. N., Promak, A. V. and Travkin, V. S., (1986). Mathematical models of flow and mass transfer in urban roughness layer, *Problemy Kontrolya i Zaschita Atmosfery ot Zagryazneniya*, No. 12, 3-10.(in Russian)

Thauvin, F. and Mohanty, K. K., 1998, Network modeling of non-Darcy flow through porous media, *Transport in Porous Media*, Vol. 31, pp. 19-37.

Thompson, K. E. and Fogler, H. S., 1997, Modeling flow in disordered packed beds from pore-scale fluid mechanics, *AIChE Journal*, Vol. 43, No. 6, pp. 1377-1389.

Travkin, V. S., Catton, I., and Gratton, L., 1993, Single phase turbulent transport in prescribed non-isotropic and stochastic porous media, HTD Vol. 240, *Heat Transfer In Porous Media*, ASME, pp. 43-48.

Travkin V. S. and Catton, I., 1995, A two temperature model for turbulent flow and heat transfer in a porous layer, *J. Fluids Engineering*, Vol. 117, pp. 181-188.

Travkin V. S. and Catton, I., 1998, Porous media transport descriptions - non-local, linear and non-linear against effective thermal / fluid properties, *Advances in Colloid and Interface Science,* Vol. 76-77, pp. 389-443.

Travkin, V. S. and Catton, I., 1999, Nonlinear effects in multiple regime transport of momentum in longitudinal capillary porous media morphology, *Journal of Porous Media*, Vol. 2, No. 3, pp. 277-294.

Travkin, V. S., Hu, K. and Catton, I., 2001, Exact Closure Procedures of Hierarchical VAT Capillary Thermo-Convective Problem for Turbulent and Laminar Regimes, International Mechanical Engineering Congress and Exposition, November 11-16, 2001, New York, NY.

Yuan, Z. G., Somerton, W. H., and Udell, K. S., 1991, Thermal Dispersion in Thick-Walled Tubes as a Model of Porous Media, *Int. J. Heat Mass Transfer*, Vol. 34, No. 11, pp. 2715-2726.

Zanotti, F. and Carbonell, R. G., 1984, Development of transport equations for multiphase systems - III, *Chemical Engineering Science*, Vol. 39, No. 2, pp. 299-311.