# RADIATION HEAT TRANSPORT IN POROUS MEDIA

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## Abstract

At present most work treats radiative transport in heterogeneous media as if it were a homogeneous medium, then relies on different methods to simulate the medium heterogeneity or uses similar governing equations with assigned distributions for coefficients. This approach is widely used although almost never found in other fields of heat and mass transport. The lack of generality in present theoretical treatments of radiative transport in heterogeneous media is addressed by rigorous development of a set of governing equations. The new rigorous equations for radiation transport in heterogeneous media are presented for the first time. As part of the development of the new set of equations for electromagnetic and spectral intensity fields, the diffusion approximation is explored.

## NOMENCLATURE

$\mathbf{B}$ - magnetic flux density [Wb/m$^2$] [1]
$c_p$ - specific heat $[J/(kg \cdot K)]$
$ds$ - interface differential area in porous medium $[m^2]$
$\partial S_{12}$ - internal surface in the REV $[m^2]$
$\mathbf{D}$ - electric flux density [C/m$^2$]
$\mathbf{E}$ - electric field [V/m]
$\widetilde{f}_i \equiv \{f_i\}_i$- VAT intrinsic phase averaged over $\Delta\Omega_i$ value $f$
$< f >_i$ - VAT phase averaged value $f$, averaged over $\Delta\Omega_i$ in a REV
$\hat{f}$ - VAT morpho-fluctuation value of $f$ in a $\Omega_i$
$I_\nu$ - radiation intensity [W/(m$^2$ sr Hz)]
$I_{\nu b}$ - spectral blackbody intensity [W/(m$^2$ sr Hz)]
$I_b$ - total blackbody radiation intensity [W/(m$^2$ sr)]
$I_{b21}$ - blackbody specific surface radiation intensity [W/(m$^2$)]
$\mathbf{j}$ - current density [A/m$^2$]
$< f >_t$ - time averaged value $f$
$k_1 = k_f$ - fluid phase thermal conductivity $[W/(mK)]$
$k_2 = k_s$ - homogeneous effective thermal conductivity of solid phase $[W/(mK)]$
$\mathbf{H}$ - magnetic field [A/m]
$m$ - porosity [-]
$\langle m \rangle$ - averaged porosity [-]
$n$ - refraction index [-]
$\mathbf{q}^r$ - radiation flux $[W/m^2]$
$p$ - phase function [-]
$\langle s_2 \rangle$ - solid phase fraction [-]
$S_{12}$ - specific surface of a porous medium $\partial S_{12}/\Delta\Omega$ $[1/m]$
$T$ - temperature $[K]$
$T_2 = T_s$ - solid phase temperature $[K]$
$\widetilde{T}^s_{ij}$ - averaged interface surface temperature when medium $i$ is in negative direction $[K]$

Subscripts.
$f \equiv 1$ - fluid phase
$s \equiv 2$ - solid phase
Superscripts.
$\sim$- value in fluid phase averaged over the $\Delta\Omega_1$
$*$ - complex conjugate variable
Greek letters.
$\widetilde{\alpha}_{21}$ - averaged heat transfer coefficient over $\partial S_{12}$ $[W/(m^2K)]$
$\beta$ - total extinction coefficient [1/m]
$\beta_\nu$ - extinction coefficient [1/m]
$\varepsilon_d$ – dielectric permittivity [Fr/m]
$\varepsilon_{ij}$ - radiative hemispherical emissivity from phase $i$ to phase $j$ with phase $i$ being into the negative direction $\mathbf{\Omega}$
$\varepsilon^r_{21}$ - total radiative hemispherical emissivity from phase 2 to phase 1 in the REV
$\lambda_j$ - prescribed Markovian transition length in medium $j$ [m]
$\mu_m$ - magnetic permeability [H/m]
$\nu$ - frequency [Hz]
$\rho$ - electric charge density [C/m$^3$] and mass density [kg/m$^3$]
$\sigma$ - Stephan-Boltzmann constant [W/(m$^2$ K$^4$)]
$\sigma_e$ - medium specific electric conductivity [A/V/m]
$\Phi$ - electric scalar potential [V]
$\psi$ - particle intensity per unit energy (frequency)
$\widetilde{\psi}^{se}_j$ - interface ensemble-averaged value of $\psi$, with phase $j$ being to the left
$\{\psi\}$ - ensemble-averaged value of $\psi$
$\omega$ - angular frequency [rad/s]
$\varkappa_{\nu a} = \varkappa_a$ - absorption coefficient [1/m]
$\varkappa_{\nu s} = \varkappa_s$ - scattering coefficient [1/m]
$\Omega$ - vector of radiation direction considered
$\Delta\Omega$ - representative elementary volume (REV) [m$^3$]
$\Delta\Omega_1 = \Delta\Omega_f$ - pore volume in a REV [m$^3$]

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$\Delta\Omega_2 = \Delta\Omega_s$ - solid phase volume in a REV [$m^3$]

# INTRODUCTION

Radiation transport problems in porous (and heterogeneous) media, including work by Tien (1988), Siegel and Howell (1992), Hendricks and Howell (1994), Kumar et al. (1990), Singh and Kaviany (1994), Tien and Drolen (1987), and Lee et al.(1994), are primarily based on governing equations resulting from the assumption of a homogeneous medium. This implicitly implies that specific problem features due to heterogeneities can be described using different methods for evaluation of the interim transport coefficients as, for example, done by Al-Nimr and Arpaci (1992), Kumar and Tien (1990), Lee (1990), Lee et al. (1994) and Dombrovsky (1996). While this kind of approach is legitimate, it presents no fundamental understanding of the processes because the governing equations are (or is) suffer from the initial assumption that strictly describes only homogeneous media. Further, it is difficult to represent hierarchical physical systems behavior with such models as will be touched on later.

Review papers like that of Reiss (1990) describe the progress in the field of dispersed media radiative transfer. The few works on heterogeneous radiative or electromagnetic transport (see Dombrovsky (1996), Adzerikho et al. (1990), van de Hulst (1981), Bohren and Huffman (1983), Lorrain and Corson (1970), Lindell et al.(1994), and Lakhtakia et al. (1989)) approaching the study of transport in disperse media with the emphasis on known scattering techniques and their improvements.

The area of neutron transport and radiative transport in heterogeneous medium being developed by Pomraning (1991, 1992, 1997), and Malvagi and Pomraning (1996) treats linear transport in a two-phase (two materials) media with stochastic coefficients. This approach is the same approach that has been used to treat thermal and electrical conductivity in heterogeneous media and to this point it has not been brought to a high enough level to include variable properties, their nonlinearities, and cross field (electrical and thermal or magnetic) phenomena.

In this work we will restrict ourselves to this brief analysis of previous work and will show that the best theoretical tool is the non-local description of hierarchical, multiscaled processes resulting from application of volume averaging theory (VAT). Application of VAT to radiative transport in porous medium is based on our previous work on thermal and momentum transport phenomena in two-phase heterogeneous media. The governing conservation equations for heat and momentum transport can be found in Travkin and Catton (1998a) and the references therein.

Recent research by Lee et al. (1994) on attenuation of electromagnetic and radiation fields in fibrous media has shown a high extinction rate for infrared radiation. The problem is treated as a scattering problem for a single two-layer cylinder by Farone and Querfeld (1966), Samaddar (1970), and Bohren and Huffman (1983). The process of radiative heat transport in porous media is very similar to propagation of electromagnetic waves in porous media and will also be evaluated. These two very close fields seems not much been considered as a coherent areas. Complicated problems of propagation of electromagnetic waves through the fiber gratings has been primarily the subject of electrodynamics. The most notable work in this area is that of Pereverzev and Ufimtsev (1994), Figotin and Kuchment (1996, 1998), Figotin and Godin (1997), Botten et al. (1998), McPhedran et al. (1996, 1997). No effort seems to have been made to translate results obtained for polarized electromagnetic radiation to the area of heat radiative transfer.

Detailed Micro-Modeling (DMM) of electromagnetic wave scattering has been based on single particles or specific arrangements of particulate media. Direct Numerical Modeling (DNM) of the problem seems enables one to do a full analysis of the fields involved. As we discuss below, the analysis of the results of a DNM is limited in the performance of a scaling analysis, which is the goal in most situations. Performing DNM without a proper scaling theory is like performing experiments, often very challenging and expensive, without proper data analysis yields an amount of detailed field results, but not the needed bulk or mean media physical characteristics.

It is obvious that DMM-DNM results can not completely meet the needs of Heterogeneous Media Modeling (HMM). The differences between DMM-DNM and Heterogeneous Media Modeling and why DMM cannot yield a sufficient description of heterogeneous media transport phenomena is answered by analysis of the following issues:

1) There is a mismatching at the boundaries (phase interfaces) leading to different boundary conditions for DMM and for the bulk (averaged characteristics) material fields.

2) The spacial scaling of heterogeneous problems with the selected Representative Elementary Volumes (REV) (for DMM) arises if one needs to address large or small deviations in the elements considered. This is particularly important if the underlying physics of some of them is different. This means that spatial heterogeneities of characteristics or morphology variation in a coordinate direction precludes use DMM.

3) Random morphologies can be treated as long as the numerical experiments provided by DMM-DNM can be translated to the same form as the overall spatial bulk characteristics modeled. This is not so simple, because the equations used are for an assumed media.

4) Discrete continuum gap closure or matching , as long as DMM-DNM actually unrealistically claims on the description and simulation of continuum phenomena. This is the most fundamental drawback when DMM is being used to apply as the most exact reflection of the real physical phe-

nomena. That means, that as soon as the developed solution of the DMM problem exists it needs to be matched to the correspondent HMM. Otherwise, DMM is only valid for the scales in which the problem was stated and in which the experimental confirmation only to be sought. Mostly it is for the regular morphologies. So, no conclusion or generalization could be developed for the next or higher levels of hierarchy of the matter description. Meaning, there is no co-junction in the description of the different physical scales.

5) Interpretation of the results of DMM and DNM are always a problem. If the results are presented for a heterogeneous continuum, the second point applies. If the results are for a particular media then how to relate them to a continuum problem of interest or even to a slightly different problem is not clear. Usually, both limits are transposed into a continuum model in the form of some differential equation or equations. If the results obtained then fit a statistical model, most often assumed to be statistical averaging, then it is only valid for the circumstances where the group of facts are taken separately as independent events. Otherwise, higher moments appear and they are almost always neglected.

6) The most sought after characteristics of a heterogeneous media are effective transport coefficients that can be expressed using conventional definitions. For example the effective conductivity for a two-phase media defined by

$$-\langle \mathbf{j} \rangle = \sigma_e^* \nabla \langle \Phi \rangle = \sigma_{e2} \nabla \langle \Phi \rangle +$$

$$+ (\sigma_{e1} - \sigma_{e2}) \frac{1}{\Delta\Omega} \int\limits_{\Delta\Omega_1} \nabla \Phi \, d\omega,$$

but in only a fraction of the problems can be expressed this way using DMM-DNM exact solutions. The issue is that in a majority of problems, having a two-field DMM-DNM exact solution is not enough to find an effective conductivity or permittivity or a mean absorption coefficient.

Most recent work on radiative transport is based on linearized radiative transfer equations for porous media. We will first review this work to set the stage for the development that will follow. This work extends our results in the theoretical advancement of fluid mechanics, heat transport and electrodynamics in heterogeneous media (Travkin et al., 1994, Catton and Travkin, 1996; Travkin and Catton, 1995, 1997, 1998a,1998b; Travkin et al., 1998, 1999a,b,c; Ryvkina et al., 1998; and Ponomarenko et al., 1999) and provides a means for formulation of radiative transport problem in porous media using heterogeneous VAT approach and electrodynamics language. Based on our previous work, a theoretical description of radiative transport in porous media is developed along with the Maxwell equations for a heterogeneous medium.

## LINEAR RADIATIVE TRANSFER EQUATIONS IN POROUS MEDIA

The equation for radiative transport in a homogeneous medium can be written in the following general form

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \nabla \cdot (\mathbf{\Omega} I_\nu) + [\varkappa_a(\mathbf{r}) + \varkappa_s(\mathbf{r})] I_\nu = \varkappa_a(\mathbf{r}) I_{\nu b}(T) +$$

$$+ \frac{1}{4\pi}\varkappa_s(\mathbf{r}) \int\limits_{4\pi} p\left(\mathbf{\Omega}' \cdot \mathbf{\Omega}\right) I_\nu\left(\mathbf{r}, \mathbf{\Omega}'\right) d\mathbf{\Omega}', \quad (1)$$

$$I_\nu = I_\nu(\mathbf{r}, \mathbf{\Omega}, t),$$

and for steady state, using the identity

$$\nabla \cdot (\mathbf{\Omega} I_\nu) = \mathbf{\Omega} \cdot \nabla I_\nu,$$

in the form

$$\mathbf{\Omega} \cdot \nabla I_\nu + [\varkappa_a(\mathbf{r}) + \varkappa_s(\mathbf{r})] I_\nu = \varkappa_a(\mathbf{r}) I_{\nu b}(T) +$$

$$+ \frac{1}{4\pi}\varkappa_s(\mathbf{r}) \int\limits_{4\pi} p\left(\mathbf{\Omega}' \cdot \mathbf{\Omega}\right) I_\nu\left(\mathbf{r}, \mathbf{\Omega}'\right) d\mathbf{\Omega}'. \quad (2)$$

In terms of a spectral source function $S_\nu(s)$ the equation can be written in a particularly simple form

$$\frac{1}{\beta_\nu}\mathbf{\Omega} \cdot \nabla I_\nu + I_\nu = S_\nu(s), \quad (3)$$

where the extinction coefficient (total cross section - Pomraning, 1996; 1997) is

$$\beta_\nu = \varkappa_a(\mathbf{r}) + \varkappa_s(\mathbf{r}).$$

Linear particle (neutron, for example) transport in heterogeneous medium is assumed by Malvagi and Pomraning (1996) and Pomraning (1997) to be described by

$$\mathbf{\Omega} \cdot \nabla \psi + \beta(\mathbf{r}) \psi = S(\mathbf{r}, \mathbf{\Omega}) +$$

$$+ \frac{1}{4\pi} \int\limits_{4\pi} \varkappa_s\left(\mathbf{r}, \mathbf{\Omega}' \cdot \mathbf{\Omega}\right) \psi\left(\mathbf{r}, \mathbf{\Omega}'\right) d\mathbf{\Omega}', \quad (4)$$

where the quantities $\beta(\mathbf{r})$, $\varkappa_s(\mathbf{r})$ and $S(\mathbf{r}, \mathbf{\Omega})$ are taken to be two-states discrete random variables. By assuming this, one needs to treat the porous (heterogeneous) medium as a binary medium which has two magnitudes for each of the random variables and a particle encounters alternating segments of medium with those magnitudes while traversing the medium. When $\beta$, $\varkappa_s$ and $S$ are assumed to be random variables, equation (4) is treated as an ensemble-averaged equation (see Malvagi and Pomraning, 1992; and Pomraning and Su, 1994),

$$\mathbf{\Omega} \cdot \nabla (p_i \{\psi_i\}) + p_i \{\beta_i\} \{\psi_i\} = p_i \{S_i\} +$$

$$+ \frac{\{\varkappa_{si}\}}{4\pi} p_i \varphi_i + \frac{p_j \widetilde{\psi}_j^{se}}{\lambda_j} - \frac{p_i \widetilde{\psi}_i^{se}}{\lambda_i}, \ i = 1, 2, \ j \neq i, \quad (5)$$

$$\varphi = \int_{4\pi} \psi \left( \mathbf{r}, \mathbf{\Omega}' \right) d\mathbf{\Omega}',$$

where $\{\psi_i\}$ is the conditional ensemble averaged function $\psi$ at some point $\mathbf{r}$ that is in phase $i$, and $\widetilde{\psi}_j^{se}$ and $\widetilde{\psi}_i^{se}$ are the interface ensemble-averaged fluxes. The solution to this equation is also supposed to be ensemble-averaged. The overall averaging over the both phases is given by

$$\langle \psi (\mathbf{r}, \mathbf{\Omega}) \rangle = p_1 \{\psi_1\} + p_2 \{\psi_2\} \tag{6}$$

where $p_1$ and $p_2$ are the probabilities of point $\mathbf{r}$ being in medium $i = 1$ or2, and $\{\psi_i\}$ is the conditional ensemble averaged value of $\psi$, when $\mathbf{r}$ is in medium $i$.

Ensemble averaging in this representation is obtained by averaging of medium features, including coefficients, along a straight line the $\mathbf{\Omega}$ direction - or by non-local 1D line averaging in terms of the physical fields considered. Most of this kind of work is related to the Markovian statistics by alternating along the line of two phases of the medium (Pomraning 1991, 1992, 1996, 1997).

The ensemble averaging procedure suggested in (5) dignifies that the two last terms in the averaged equation reflects the finite correlation length (interconnection) in a single nonlinear term $\beta (\mathbf{r}) \psi$. This kind of averaging result in very simple closure statements derived using hierarchical volume averaging theory procedures, as will be shown below. A major problem in using ensemble averaging techniques is that the processes and phenomena going at each separate site within separate elements of the heterogeneous media can not be resolved completely with the purely statistical approach of ensemble averaging.

To make an ensemble averaging method workable, researchers always need to formulate the final problem or solution in terms of spacially specific statements or in terms of the original spacial Volume Averaging Theory (VAT). Examples of this are numerous, see the review by Buyevich and Theofanous (1997).

## NON-LOCAL VOLUME AVERAGED RADIATIVE TRANSFER EQUATIONS

The basis for the development in this work is called volume averaging theory (VAT). We will present some aspects VAT that are now becoming well understood and have seen substantial progress in thermal physics and in fluid mechanics. The need for a method that enables one to develop general, physically based models of a group of physical objects (for example, molecules, atoms, crystals, phases, etc.) that can be substantiated by data (statistical or analytical in nature) is clear. In modern physics it is usually accomplished using statistical data and theoretical methods. One of the major drawbacks of this widely used approach is that it does not give a researcher the capability to relate the spacial and morphological parameters of a group of objects to the phenomena of interest when it is described at the upper level of the hierarchy. Often the equations obtained by these methods differ from one another even when describing the same physical phenomena.

The drawbacks of existing methods do not arise when the VAT mathematical approach is used. At the present time, there is an extensive literature and many books on linear, homogeneous and layered system electromagnetic and acoustic wave propagation (Adzerikho et al., 1990; Bohren and Huffman, 1983; Dombrovsky, 1996; Lindell et al., 1994; Lakhtakia et al., 1989; Lorrain and Corson, 1970; Siegel and Howell, 1992; van de Hulst, 1981). It is surprising that these phenomena are often described by almost identical mathematical statements and governing equations for both heterogeneous and homogeneous media.

Major developments in the use of VAT, showing the potential for application to the electrophysical and acoustics phenomena in heterogeneous media, are found in Travkin and Catton (1998b), Travkin et al. (1998, 1999a,b,c), experimental applications in Ryvkina et al. (1998) and Ponomarenko et al. (1999). It has been demonstrated during the past twenty years of VAT based modeling in the thermal physics and fluid mechanics area (see, Slattery, 1980; Whitaker, 1997; Kaviany, 1995; Gray et al. 1993) that the potential of the approach are enormous. Substantial success has also been achieved in analyzing the more narrow phenomena of electromagnetic wave propagation in porous media.

We consider here radiative transfer in porous media using a hierarchical approach to describe physical phenomena in a heterogeneous media. The physical features of lowest scale of the medium considered and their averaged characteristics are obtained using special mathematical instruments for describing hierarchical processes, namely VAT. At the upper (next) level of the hierarchy, physical phenomena have the physical medium point-wise characteristics resulting from averaged lower scale characteristics.

The subvolume $\Delta\Omega$ inside the domain where the problem is stated and over which functions are averaged is called the representative elementary volume (REV), see for detailed description Slattery (1980), Whitaker (1997), Gray et al. (1993), . It is assumed that the heterogeneous media is two phase with known spacial morphology, or at least some features of the morphology are known, and that the generally random volume fraction of the phase 1 can be expresses as additive components. The average value of $\left\langle m_1(\vec{x}) \right\rangle$ in the REV and its fluctuations in various directions can then be expressed

$$m_1(\vec{x}) = \left\langle m_1(\vec{x}) \right\rangle + \widehat{m}_1(\vec{x}), \qquad \langle m_1 \rangle = \frac{\Delta\Omega_1}{\Delta\Omega}.$$

Five types of averaging over the REV function $f$ are defined by the following averaging operators [2, 11]

$$\langle f\rangle = \langle f\rangle_1 + \langle f\rangle_2 = \langle m_1\rangle \widetilde{f}_1 + (1 - \langle m_1\rangle)\, \widetilde{f}_2, \qquad (7)$$

with the phase average for first phase being

$$\langle f\rangle_1 = \langle m_1\rangle \frac{1}{\Delta\Omega_1} \int\limits_{\Delta\Omega_1} f\left(t, \vec{x}\right) d\omega = \langle m_1\rangle \widetilde{f}_1, \qquad (8)$$

The intraphase average is

$$\{f\}_1 = \widetilde{f}_1 = \frac{1}{\Delta\Omega_1} \int\limits_{\Delta\Omega_1} f\left(t, \vec{x}\right) d\omega, \qquad (9)$$

where $\widetilde{f}_1$ is an average over the phase 1 volume $\Delta\Omega_1$, $\widetilde{f}_2$ is an average over the second phase volume $\Delta\Omega_2$=$\Delta\Omega$ - $\Delta\Omega_1$, and $\langle f\rangle$ is an average over the whole REV. In accordance with one of the major averaging theorems, averaging the $\nabla$ operator Whitaker (1997), Gray et al. (1993), the average operator $\nabla$ becomes

$$\langle \nabla f\rangle_1 = \nabla \langle f\rangle_1 + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} f\, \vec{ds}_1 \; . \qquad (10)$$

The differentiation theorem for the intraphase averaged function is

$$\begin{aligned} \{\nabla f\}_1 &= \nabla \widetilde{f} + \frac{1}{\Delta\Omega_f} \int\limits_{\partial S_{12}} \widehat{f}\, \vec{ds}_1, \\ \widehat{f} &= f - \widetilde{f}, \quad f \;\forall\; \Delta\Omega_1, \end{aligned} \qquad (11)$$

where $\partial S_{12}$ is the inner surface in the REV, $\vec{ds}$ is the second-phase, inward-directed differential area in the REV ( $\vec{ds} = \vec{n}\; dS$). The same kind of operator involving **rot** will result in the following averaging theorems

$$\langle \nabla \times \mathbf{f}\rangle_1 = \nabla \times \langle \mathbf{f}\rangle_1 + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \vec{ds}_1 \times \mathbf{f}, \qquad (12)$$

$$\{\nabla \times \mathbf{f}\}_1 = \nabla \times \{\mathbf{f}\}_1 + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \vec{ds}_1 \times \widehat{\mathbf{f}}. \qquad (13)$$

More detail on the operations and rules induced by heterogeneity in VAT can be found in Whitaker (1997), Kaviany (1995) and elements of nonlinear operators and equation treatment in work by Primak et al. (1986), Catton and Travkin (1986), Travkin and Catton (1998a). Rigorous application to linear and nonlinear electrodynamics and electrostatic problems is described in Travkin et al. (1999a,b).

The phase averaging the equation for linear local thermal equilibrium radiative transfer,

$$\begin{aligned} &\mathbf{\Omega} \cdot \nabla I_\nu + \beta_\nu (\mathbf{r})\, I_\nu = \varkappa_a (\mathbf{r})\, I_{\nu b} (T) + \\ &+ \frac{1}{4\pi} \varkappa_s (\mathbf{r}) \int\limits_{4\pi} p\left(\mathbf{\Omega}' \cdot \mathbf{\Omega}\right) I_\nu \left(\mathbf{r}, \mathbf{\Omega}'\right) d\mathbf{\Omega}', \end{aligned} \qquad (14)$$

in phase 1 yields the following VAT radiative equation (VARE)

$$\begin{aligned} &\mathbf{\Omega} \cdot \left[\nabla \langle I_{\nu 1}\rangle_1 + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} I_{\nu 1}\, \vec{ds}_1\right] + \left\langle \widetilde{\beta}_{\nu 1} \widetilde{I}_{\nu 1}\right\rangle_1 = \\ &= \langle \varkappa_{a1} (\mathbf{r})\, I_{\nu b1} (T)\rangle_1 + \\ &+ \frac{\varkappa_{s1}}{4\pi} \langle \varphi_1\rangle_1 - \left\langle \widehat{\beta}_{\nu 1} \widehat{I}_{\nu 1}\right\rangle_1, \; i = 1, \\ &\varphi = \int\limits_{4\pi} p\left(\mathbf{\Omega}' \cdot \mathbf{\Omega}\right) I_\nu \left(\mathbf{r}, \mathbf{\Omega}'\right) d\mathbf{\Omega}', \end{aligned} \qquad (15)$$

when it is assumed that $\varkappa_{si}$ is a constant as done by Malvagi and Pomraning (1992), Pomraning and Su, 1994, and others.

The additional terms appearing in the VARE in some instances are similar but in others they have a different interpretation in the ensemble averaged equation (5). For example the term

$$-\left\langle \widehat{\beta}_{\nu 1} \widehat{I}_{\nu 1}\right\rangle_1, \qquad (16)$$

in (15) is the result of fluctuations correlation inside of medium 1 in the REV but it is described by

$$\frac{p_j \widetilde{\psi}_j^{se}}{\lambda_j} - \frac{p_i \widetilde{\psi}_i^{se}}{\lambda_i}, \qquad (17)$$

in Malvagi and Pomraning (1992) as it is an exchange of energy term between the two phases across the interface surface area $\partial S_{12}$. Because ensemble averaging methodologies in Malvagi and Pomraning (1992) does not treat nonlinear terms very well and incorrectly averages differential operators like $\nabla$, for example, terms do not appear in equation (5) that reflect the interface flux exchange. In VARE, equation (15), the interface exchange term naturally appears as a result of averaging the $\nabla$ operator

$$\mathbf{\Omega} \cdot \left(\frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} I_{\nu 1}\, \vec{ds}_1\right). \qquad (18)$$

When the coefficients in the radiative transfer equation are dependant functions, more linearized terms are observed in the corresponding VARE

$$\begin{aligned} &\mathbf{\Omega} \cdot \nabla \langle I_{\nu 1}\rangle_1 + \left\langle \widetilde{\beta}_{\nu 1} \widetilde{I}_{\nu 1}\right\rangle_1 = \left\langle \widetilde{\varkappa}_{a1} \widetilde{I}_{\nu b1} (T)\right\rangle_1 + \\ &+ \left\langle \widehat{\varkappa}_{a1} \widehat{I}_{\nu b1}\right\rangle_1 + \frac{1}{4\pi} \left(\langle \widetilde{\varkappa}_{s1} \widetilde{\varphi}_1\rangle_1 + \langle \widehat{\varkappa}_{s1} \widehat{\varphi}_1\rangle_1\right) - \\ &- \mathbf{\Omega} \cdot \left(\frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} I_{\nu 1}\, \vec{ds}_1\right) - \left\langle \widehat{\beta}_{\nu 1} \widehat{I}_{\nu 1}\right\rangle_1, \; i = 1, \end{aligned} \qquad (19)$$

while continuing to threat the emissivity as via the Planck's function. This equation should be accompanied by the VAT heat transfer equations in both porous medium phases (see, for example, Travkin and Catton, 1998).

The heat transport within solid phase 2, combining conductive and possible radiative transfer is described by

$$\langle s_2\rangle (\varrho c_p)_2 \frac{\partial \widetilde{T}_2}{\partial t} = k_2\nabla^2\left(\langle s_2\rangle \widetilde{T}_2\right) +$$

$$+k_2\nabla\cdot\left[\frac{1}{\Delta\Omega}\int\limits_{\partial S_{12}} T_2\,\vec{ds_2}\right] +$$

$$+\frac{k_2}{\Delta\Omega}\int\limits_{\partial S_{12}} \nabla T_2\cdot\vec{ds_2} +\nabla\cdot\langle \mathbf{q}^r\rangle_2 + \frac{1}{\Delta\Omega}\int\limits_{\partial S_{12}} \mathbf{q}^r\cdot\vec{ds_2}\,. \quad (20)$$

The third and fifth terms on the r.h.s. model the heat exchange rate between the phases. In an optically thick medium, for example, the radiation flux term written in terms of the total blackbody radiation intensity is

$$\nabla\cdot\langle \mathbf{q}^r\rangle_2 = \nabla\cdot\left\langle -\frac{4}{3\beta}\nabla\left(I_b\right)\right\rangle_2 =$$

$$= \nabla\cdot\left\langle -\frac{4}{3\beta}\nabla\left(\frac{n^2\sigma T^4}{\pi}\right)\right\rangle_2, \quad (21)$$

where $\beta$ is the total extinction coefficient. An energy equation similar to equation (20) needs to be written for the fluid filled volume, phase 1 of the porous medium. The radiation flux term would be much more complex because of the spectral characteristics of radiation in a fluid.

Closure is needed for the second, third and fifth terms in equation (20) on the r.h.s. For convective heat exchange, the last term can be written

$$\frac{k_2}{\Delta\Omega}\int\limits_{\partial S_{12}} \frac{\partial T}{\partial x_i}\cdot\vec{ds_2} = \widetilde{\alpha}_{21}\, S_{12}\,\left(\{T\}_1 - \{T\}_2\right), \quad (22)$$

by noting that

$$\frac{1}{\Delta\Omega}\int\limits_{\partial S_{12}} k_2\,\frac{\partial T}{\partial x_i}\cdot\vec{ds_2} = -\frac{1}{\Delta\Omega}\int\limits_{\partial S_{12}} k_2\,\frac{\partial T}{\partial \mathbf{n}_1}ds\cdot\mathbf{n}_1 =$$

$$= \frac{1}{\Delta\Omega}\int\limits_{\partial S_{12}} \mathbf{q}_1\cdot\vec{ds_1} = \widetilde{\alpha}_{21}\, S_{12}\,\left(\{T\}_1 - \{T\}_2\right). \quad (23)$$

This type of closure procedure is appropriate for description of fluid-solid media heat exchange and has been considered by many as an analog for solid-solid heat exchange. A more strict and precise integration of the heat flux over the interface surface using the IVth kind of boundary conditions, gives the exact closure for the term in the governing equations for the neighboring phase. This would be an adequate solution for the portion of heat exchange by conduction to and from the fluid phase, a conjugate problem.

The radiative energy exchange across the interface surface is difficult to formulate because of its spectral characteristics and the boundary conditions that must be satisfied. When the fluid phase is assumed to be optically thin, an approximate closure expression results

$$\frac{1}{\Delta\Omega}\int\limits_{\partial S_{12}} \mathbf{q}^r\cdot\vec{ds_2} = \frac{1}{\Delta\Omega}\int\limits_{\partial S_{12}} \left(\frac{\sigma\left(T_{12}^4 - T_{21}^4\right)}{\left(\frac{1}{\varepsilon_{12}}+\frac{1}{\varepsilon_{21}}-1\right)}\right)\cdot\vec{ds_2} \cong$$

$$\cong \left(\frac{\sigma\left(\left(\widetilde{T}_{12}^s\right)^4 - \left(\widetilde{T}_{21}^s\right)^4\right)S_{12}}{\left(\frac{1}{\varepsilon_{12}}+\frac{1}{\varepsilon_{21}}-1\right)}\right), \quad (24)$$

using an interpretation of the averaged surface temperatures on opposite sides of the interface developed by Malvagi and Pomraning (1992). Another approximation is justifiable for an optically thick fluid phase. It uses the specific blackbody surface radiation intensity $I_{b21} = n^2\sigma T_{21}^4$ to close the integral energy exchange term as follows

$$\frac{1}{\Delta\Omega}\int\limits_{\partial S_{12}} \mathbf{q}^r\cdot\vec{ds_2} \cong \frac{1}{\Delta\Omega}\int\limits_{\partial S_{12}} \left(n^2\sigma T_{21}^4\right)\vec{ds_2} \cong$$

$$\cong \varepsilon_{21}^r\sigma\left(\widetilde{T}_{21}^s\right)^4 S_{12}, \quad (25)$$

where $\varepsilon_{21}^r$ is the total radiative hemispherical emissivity from phase 2 to phase 1 in the REV.

The closure of equations (5) is accomplished by assuming equality (Malvagi and Pomraning, 1992; Pomraning and Su, 1994) between the interface surface and ensemble (1D in this case) averaged functions

$$\widetilde{\psi}_j^{se} = \{\psi_i\}, \quad (26)$$

as was done in heat and mass transfer porous medium problems see, for example (Crapiste et al., 1986).

## VAT MAXWELL'S EQUATIONS IN POROUS MEDIUM

Radiative transport in porous media is propagation of electromagnetic waves in heterogeneous media. As such, the nonlocal VAT based electrodynamic equations for heterogeneous media used here might have coefficients that can be inhomogeneous or nonlinear functions. The interface surface is taken to be immobile. Inhomogeneity is assumed in a form that involves space-function dependent smooth coefficients. The electrodynamic equations for that kind of an inhomogeneous nonferromagnetic substance are

$$\nabla\cdot(\varepsilon_d\mathbf{E}) = \rho,\ \nabla\cdot(\mu_m\mathbf{H}) = 0, \quad (27)$$

$$\nabla\times\mathbf{E} = -\frac{\partial}{\partial t}\left(\mu_m\mathbf{H}\right), \quad (28)$$

$$\nabla\times\mathbf{H} = \frac{\partial}{\partial t}\left(\varepsilon_d\mathbf{E}\right) + \mathbf{j}, \quad (29)$$

(without an external field induced current density vector $\mathbf{j}^{(e)}$ ) with constitutive relations

$$\mathbf{B} = \mu_m \mathbf{H},\ \mathbf{D} = \varepsilon_d \mathbf{E},\ \mathbf{j} = \sigma_e \mathbf{E}. \tag{30}$$

Averaging of the first and second equations (27) is done using the left hand side as the nonlinear operator. After averaging over the phase 1 using $\langle\rangle_1$ , equation (27) becomes

$$\nabla \cdot \left[\langle m_1\rangle \widetilde{\varepsilon}_{d1} \widetilde{\mathbf{E}}_1\right] + \nabla \cdot \left[\langle m_1\rangle \left\{\widehat{\varepsilon}_{d1} \widehat{\mathbf{E}}_1\right\}_1\right] +$$

$$+\frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \left(\varepsilon_{d1}\mathbf{E}_1\right) \cdot \vec{ds_1} = \langle\rho\rangle_1 , \tag{31}$$

and the second averaged electric field equation is

$$\nabla \times \left(\langle m_1\rangle \widetilde{\mathbf{E}}_1\right) + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \vec{ds_1} \times \mathbf{E}_1 = -\frac{\partial}{\partial t} \langle \mu_m \mathbf{H}\rangle_1 . \tag{32}$$

The averaged magnetic field equations are

$$\nabla \cdot \left(\langle m_1\rangle \widetilde{\mu}_{m1} \widetilde{\mathbf{H}}_1\right) + \nabla \cdot \left[\langle m_1\rangle \left\{\widehat{\mu}_{m1} \widehat{\mathbf{H}}_1\right\}_1\right] +$$

$$+\frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \left(\mu_{m1}\mathbf{H}_1\right) \cdot \vec{ds_1} = 0, \tag{33}$$

and

$$\nabla \times \left(\langle m_1\rangle \widetilde{\mathbf{H}}_1\right) + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \vec{ds_1} \times \mathbf{H}_1 =$$

$$= \frac{\partial}{\partial t} \langle \varepsilon_d \mathbf{E}\rangle_1 + \left[\langle m_1\rangle \widetilde{\sigma}_{e1} \widetilde{\mathbf{E}}_1 + \langle m_1\rangle \left\{\widehat{\sigma}_{e1} \widehat{\mathbf{E}}_1\right\}_1\right]. \tag{34}$$

Analogous averaged equations result for the 2nd phase. More straightforward and simpler are the VAT homogeneous electrostatic equations, like for the phase 1 when the medium (constitutive) coefficients - magnetic permeability $\mu_{m1}$, dielectric permittivity $\varepsilon_{d1}$ and its conductivity $\sigma_{e1}$ are constant values, then equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_d}, \tag{35}$$

$$\nabla \times \mathbf{E} = \mathbf{0}, \tag{36}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{37}$$

$$\nabla \times \mathbf{H} = \sigma_e \mathbf{E}, \tag{38}$$

after averaging became (in the 1st phase)

$$\nabla \cdot \left(\langle m_1\rangle \widetilde{\mathbf{E}}_1\right) + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \mathbf{E}_1 \cdot \vec{ds_1} = \frac{\langle\rho\rangle_1}{\varepsilon_{d1}}, \tag{39}$$

$$\nabla \times \left(\langle m_1\rangle \widetilde{\mathbf{E}}_1\right) + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \vec{ds_1} \times \mathbf{E}_1 = 0, \tag{40}$$

$$\nabla \cdot \left[\langle m_1\rangle \widetilde{\mathbf{H}}_1\right] + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \mathbf{H}_1 \cdot \vec{ds_1} = 0, \tag{41}$$

$$\nabla \times \left(\langle m_1\rangle \widetilde{\mathbf{H}}_1\right) + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \vec{ds_1} \times \mathbf{H}_1 = \langle m_1\rangle \sigma_{e1} \widetilde{\mathbf{E}}_1. \tag{42}$$

## RADIATION TRANSPORT IN HETEROGENEOUS MEDIA USING HARMONIC ELECTROMAGNETIC FIELD MAXWELL EQUATIONS

Representing the electromagnetic field components with time-harmonic components results in

$$\nabla \cdot (\varepsilon_d \mathbf{E}) = \rho,\ \nabla \cdot (\mu_m \mathbf{H}) = 0, \tag{43}$$

$$\nabla \times \mathbf{E} = -\mathbf{i}\omega\mu_m \mathbf{H},\ \nabla \times \mathbf{H} = \mathbf{i}\omega\overline{\varepsilon}_d \mathbf{E}, \tag{44}$$

where $\overline{\varepsilon}_d$ is the complex dielectric function defined by $\overline{\varepsilon}_d = \varepsilon_d - \mathbf{i}\left(\sigma_e/\omega\right)$, and $\varepsilon_d = \varepsilon_d\left(\vec{\mathbf{x}}\right)$, $\sigma_e = \sigma_e\left(\vec{\mathbf{x}}\right)$, $\mu_m = \mu_m\left(\vec{\mathbf{x}},\omega\right)$, $\overline{\varepsilon}_d = \overline{\varepsilon}_d\left(\vec{\mathbf{x}},\omega\right)$ . In many contemporary applications the spatial dependency of these functions is neglected. Electrophysical coefficients often need to be treated as nonlinear. For example, the dielectric function can depends on $\mathbf{E}$ and $\varepsilon_d = \varepsilon_d\left(\vec{\mathbf{x}},\mathbf{E}\right)$ . The wave formulation of the Maxwell equations with constant phase coefficients for the magnetic field is

$$\nabla^2\mathbf{H} - \mu_m\sigma_e\frac{\partial \mathbf{H}}{\partial t} - \mu_m\varepsilon_d\frac{\partial^2 \mathbf{H}}{\partial t^2} = 0, \tag{45}$$

while the electric field wave equation is almost the same

$$\nabla^2\mathbf{E} - \mu_m\sigma_e\frac{\partial \mathbf{E}}{\partial t} - \mu_m\varepsilon_d\frac{\partial^2 \mathbf{E}}{\partial t^2} = \boldsymbol{\nabla}\left(\frac{\rho}{\varepsilon}\right). \tag{46}$$

Another form of the equation for $\mathbf{E}$ appears in Cartesian coordinates when electromagnetic fields are time-harmonic functions

$$\nabla^2\mathbf{E} + k^2\mathbf{E} = 0, \tag{47}$$

where the inhomogeneous function $k^2 = \omega^2\mu_m\varepsilon_d$ is the wave number squared. This equation is often applicable to linear acoustics phenomena. This category of equations can be transformed to a form legitimate for application to heterogeneous media problems.

The time-harmonic forms of equations for **rot** of electromagnetic fields are

$$\nabla \times \left(\langle m_1\rangle \widetilde{\mathbf{E}}_1\right) + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \vec{ds_1} \times \mathbf{E}_1 =$$

$$= -\mathbf{i}\omega \left[\langle m_1\rangle \widetilde{\mu}_{m1} \widetilde{\mathbf{H}}_1 + \langle m_1\rangle \left\{\widehat{\mu}_{m1} \widehat{\mathbf{H}}_1\right\}_1\right], \tag{48}$$

$$\nabla \times \left( \langle m_1 \rangle \widetilde{\mathbf{H}}_1 \right) + \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \vec{ds_1} \times \mathbf{H}_1 =$$

$$= \mathbf{i}\omega \left[ \langle m_1 \rangle \widetilde{\widetilde{\varepsilon}}_{d1} \widetilde{\mathbf{E}}_1 + \langle m_1 \rangle \left\{ \widehat{\widetilde{\varepsilon}}_{d1} \widehat{\mathbf{E}}_1 \right\}_1 \right], \qquad (49)$$

The magnetic field wave form equation with constant coefficients, when averaged over the phase 1, transforms to

$$\nabla^2 \left( \langle m_1 \rangle \widetilde{\mathbf{H}}_1 \right) + \nabla \cdot \left[ \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \mathbf{H}_1 \, \vec{ds_1} \right] +$$

$$+ \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \nabla \mathbf{H}_1 \cdot \vec{ds_1} = \mu_m \sigma_e \frac{\partial \langle \mathbf{H} \rangle_1}{\partial t} + \mu_m \varepsilon_d \frac{\partial^2 \langle \mathbf{H} \rangle_1}{\partial t^2}, \quad (50)$$

and the electric field wave equation (46) becomes

$$\nabla^2 \left( \langle m_1 \rangle \widetilde{\mathbf{E}}_1 \right) + \nabla \cdot \left[ \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \mathbf{E}_1 \, \vec{ds_1} \right] +$$

$$+ \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \nabla \mathbf{E}_1 \cdot \vec{ds_1} = \mu_m \sigma_e \frac{\partial \langle \mathbf{E} \rangle_1}{\partial t} + \mu_m \varepsilon_d \frac{\partial^2 \langle \mathbf{E} \rangle_1}{\partial t^2} +$$

$$+ \frac{1}{\varepsilon_d} \nabla \left( \langle m_1 \rangle \widetilde{\rho}_1 \right) + \frac{1}{\varepsilon_d \Delta\Omega} \int\limits_{\partial S_{12}} \rho_1 \, \vec{ds_1} \, . \qquad (51)$$

An analogous form of the averaged equation is obtained for the time-harmonic electrical field

$$\nabla^2 \left( \langle m_1 \rangle \widetilde{\mathbf{E}}_1 \right) + \nabla \cdot \left[ \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \mathbf{E}_1 \, \vec{ds_1} \right] +$$

$$+ \frac{1}{\Delta\Omega} \int\limits_{\partial S_{12}} \nabla \mathbf{E}_1 \cdot \vec{ds_1} +$$

$$+ \langle m_1 \rangle k^2 \widetilde{\mathbf{E}}_1 = 0. \qquad (52)$$

It is the naturally appearing feature of the heterogeneous media electrodynamics equations as the terms reflecting phenomena on the interface surface $\partial S_{12}$, and that fact is to be used to incorporate morphologically precise polarization phenomena as well as tunneling into heterogeneous electrodynamics, as this is being done in fluid mechanics and heat transport (Travkin and Catton, 1997, 1998; Catton and Travkin, 1996).

Using the orthogonal locally calculated directional fields $\widetilde{E}_{l1}$ and $\widetilde{E}_{r1}$ of averaged electrical field $\widetilde{\mathbf{E}}_1$, one can seek the Stokes parameters $I$, $Q$, $U$, $V$

$$I = \left\langle \widetilde{E}_{l1} \widetilde{E}^*_{l1} \right\rangle_t + \left\langle \widetilde{E}_{r1} \widetilde{E}^*_{r1} \right\rangle_t, \qquad (53)$$

$$Q = \left\langle \widetilde{E}_{l1} \widetilde{E}^*_{l1} \right\rangle_t - \left\langle \widetilde{E}_{r1} \widetilde{E}^*_{r1} \right\rangle_t, \qquad (54)$$

$$U = \mathrm{Re} \left[ \left\langle 2 \widetilde{E}_{l1} \widetilde{E}^*_{r1} \right\rangle_t \right], \qquad (55)$$

$$V = \mathrm{Im} \left[ \left\langle 2 \widetilde{E}_{l1} \widetilde{E}^*_{r1} \right\rangle_t \right], \qquad (56)$$

which characterize the intensity of polarized radiation in porous medium.

## SUMMARY

The hierarchical approach applied to radiative transfer in porous medium and to the electrodynamics governing equations (Maxwell's Equations) in heterogeneous medium yielded new volume averaged radiative transfer equations - VAREs. These equations have additional terms reflecting the influence of interfaces and inhomogeneities on radiation intensity in a porous medium and, when solve, will allow one to relate the lower scale parameters to the upper scale material behavior. The general nature of this result makes it applicable to any subatomic particle transport, including neutron transport, as well as radiative transport in heterogeneous media field.

Direct closure based on theoretical and numerical developments that have been developed for thermal, momentum and mass transport processes in a specific random porous and composite media established a basis for closure modeling in problems of radiative and electromagnetic phenomena.

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