

**b. Flow Resistance Model 2** The second flow resistance model reflects the addition of the fluid fluctuation term  $UMC_1$ :

$$\begin{aligned} -c_{d2}(\bar{U}, M_{\Omega}, \mathbf{x}) \left( \frac{S_w(x) \tilde{U}^2(x)}{2} \right) &= - \left( c_f + c_{dp} \frac{S_{wp}}{S_w} + F_2 \right) \left( \frac{S_w \tilde{U}^2}{2} \right) \\ &= (-c_{dp} S_{wp}(x) - c_{fL}(x) S_{wL}(x)) \frac{\tilde{U}^2(x)}{2} - \left( S_{wL}(x) \frac{\tilde{U}^2}{2} \right) F_2 \\ &= - \frac{1}{\rho_f \Delta \Omega} \int_{\partial S_2} p \bar{d}s + \frac{v}{\Delta \Omega} \int_{\partial S_w} \frac{\partial U}{\partial x_i} \cdot \bar{d}s - \frac{\partial}{\partial x} \langle \hat{u} \hat{u} \rangle_f. \end{aligned} \quad (266)$$

**c. Flow Resistance Model 3** The third flow resistance model reflects all of the terms responsible for momentum resistance in a porous medium:

$$\begin{aligned} c_{d3}(\bar{U}, M_{\Omega}, \mathbf{x}) \left( \frac{S_w(x) \tilde{U}^2(x)}{2} \right) &= \left( S_w(x) \frac{\tilde{U}^2}{2} \right) R_M + c_{d1}(\bar{U}, M_{\Omega}) \left( \frac{S_w \tilde{U}^2}{2} \right) \\ &= \langle m \rangle \bar{U} \frac{\partial \tilde{U}}{\partial x} - v \frac{\partial}{\partial x} \left( \frac{\partial \langle m \rangle \tilde{U}}{\partial x} \right) \\ &\quad + \frac{1}{\rho_f \Delta \Omega} \int_{\partial S_w} p \bar{d}s - \frac{v}{\Delta \Omega} \int_{\partial S_w} \frac{\partial U}{\partial x_i} \cdot \bar{d}s + \frac{\partial}{\partial x} \langle \hat{u} \hat{u} \rangle_f, \end{aligned} \quad (267)$$

where

$$\begin{aligned} \left( S_w(x) \frac{\tilde{U}^2}{2} \right) R_M &= (F_1 + F_2 + F_3) \left( S_w(x) \frac{\tilde{U}^2}{2} \right) \\ &= \langle m \rangle \bar{U} \frac{\partial \tilde{U}}{\partial x} + \frac{\partial}{\partial x} [\langle m \rangle \langle \hat{u} \hat{u} \rangle_f] - v \frac{\partial}{\partial x} \left[ \frac{\partial \langle m \rangle \tilde{U}}{\partial x} \right]. \end{aligned} \quad (268)$$

Using the notation developed earlier for the terms in the momentum equation (264) leads to a form for each of the flow resistance models that properly reflects their completeness,

$$c_{d2}(\bar{U}, M_{\Omega}, \mathbf{x}) = (UMP_1 - UMF_1) \left/ \left( \frac{S_w(x) \tilde{U}^2(x)}{2} \right) \right. \quad (269)$$

$$c_{d3}(\bar{U}, M_{\Omega}, \mathbf{x}) = (UMP_1 - UMF_1 + UMC_1) \left/ \left( \frac{S_w(x) \tilde{U}^2(x)}{2} \right) \right. \quad (270)$$

$$\begin{aligned} c_{d1}(\bar{U}, M_{\Omega}, \mathbf{x}) &= (UC_1 - UD_1 + UMP_1 - UMF_1 + UMC_1) \left/ \left( \frac{S_w(x) \tilde{U}^2(x)}{2} \right) \right. \end{aligned} \quad (271)$$

Each of the different forms will yield a correlation of a given set of data. The problem is that the effects of the different characteristics that are manifested in the terms in the equations are lost from consideration. If predictive tools are to be developed, consideration must be given to the impact of the details that the terms reflect.

### 3. Scaling in Pressure Loss Experiments and Data Analysis

Direct use of any Ergun type friction factor in a Fanning or Darcy friction factor correlation is incorrect. Ergun [167] suggested two types of friction factors, one of which is the so-called kinetic energy friction factor  $f_{ker}$ , which differs from the Fanning friction factor by a factor of three for the same medium:

$$f_f = \frac{d_h}{2\rho_f \bar{u}^2} \left( \frac{\Delta P}{L} \right) = \frac{f_{ker}}{3}. \quad (272)$$

For the same reason, direct implementation of the correlations given by Kays and London [172] should be treated with care. For example, the correlations for friction factor (Fanning) given by Kays and London for flow through an infinite randomly stacked, woven-screen matrix uses surface porosity  $p$ , and specific surface  $\alpha$  [1/m] to define a hydraulic radius  $r_h$ ,

$$r_h = \frac{p}{\alpha} = \frac{\langle m \rangle_s}{S_w}.$$

Here the specific surface  $S_w$  is defined as the interface surface divided by the volume of the REV. Unfortunately, the surface porosity  $\langle m \rangle_s$  and volume porosity  $\langle m \rangle$  are not of the same value and even if they were, the expression differs from that found earlier by a factor of 2.

Bird *et al.* [173] used the ratio of the "volume available for flow" to the "cross section available for flow" in their derivation of hydraulic radius  $r_{hb}$ . This assumption led them to the formula

$$r_{hb} = \frac{\langle m \rangle d_p}{6(1 - \langle m \rangle)}. \quad (273)$$

It would be double this value if a consistent definition were used for all systems,

$$d_h = \frac{4\langle m \rangle}{S_w} = \frac{4\langle m \rangle}{a_v(1 - \langle m \rangle)} = \frac{2\langle m \rangle}{3(1 - \langle m \rangle)} d_p = 4r_{hb}, \quad (274)$$

where  $a_v$  is the "particle specific surface" (the total particle surface area divided by the volume of the particle), and

$$S_w = a_v(1 - \langle m \rangle). \quad (275)$$

The expression given by (274) is justified when an equal or mean particle diameter is

$$d_p = \frac{6}{a_v},$$

which is the exact equation for spherical particles and is often used as substitution for granular media particles. The value of hydraulic radius given by Bird *et al.* [173], (273), was chosen by Chhabra [174] and was used in determining the specific friction factor in capillary media.

Media of globular morphologies can be described in terms of  $S_w$ ,  $\langle m \rangle$ , and  $d_p$  and can generally be considered to be spherical particles with

$$S_w = \frac{6(1 - \langle m \rangle)}{d_p}, \quad d_h = \frac{2}{3} \frac{\langle m \rangle}{(1 - \langle m \rangle)} d_p. \quad (276)$$

This expression has the same dependency on equivalent pore diameter as found for a one-diameter capillary morphology, leading naturally to

$$S_w = \frac{6(1 - \langle m \rangle)}{d_p} = \frac{6(1 - \langle m \rangle)}{\left(\frac{3}{2} \frac{(1 - \langle m \rangle)}{\langle m \rangle} d_h\right)} = \frac{4\langle m \rangle}{d_h}. \quad (277)$$

This observation leads to defining a simple "universal" porous medium scale,

$$d_h = d_{por} = \frac{4\langle m \rangle}{S_w}, \quad (278)$$

that meets the needs of both major morphologies, capillary and globular. A large amount of data exists that demonstrates the insufficiencies of the Ergun drag resistance correlation (287). Because it was developed for a specific morphology, a globular "granular" medium, application of the Ergun correlation to a medium with arbitrary relationships between porosity  $\langle m \rangle$ , specific surface  $S_w$ , and pore (particle) diameter  $d_h$  can lead to large errors.

The particle diameter  $d_p$  is often used as a length scale when reducing experimental data. Chhabra [174], for example, writes the friction factor

$$f_{cb} = \frac{d_p}{\langle m \rangle^2 \rho_f \bar{u}^2} \frac{\Delta P}{L}, \quad (279)$$

This friction factor can be related to the friction factor  $f_b$ , given by Eq. (6.4-1) of Bird *et al.* [173], to the Fanning friction factor  $f_f$ , and to the Ergun kinetic energy friction factor  $f_{ker}$  as follows:

$$f_{cb} = 2f_b = f_{ker} \left( \frac{1 - \langle m \rangle}{\langle m \rangle^3} \right) = f_f 3 \left( \frac{1 - \langle m \rangle}{\langle m \rangle^3} \right). \quad (280)$$

These models all use different length scales, leading to large uncertainties and confusion when a correlation must be selected for a particular application. Little attention is paid to these differences, often requiring new experimental data for a new medium configuration.

Only a few of the many issues important to modeling of pressure loss in porous media are addressed here. As it is known, the two-term quadratic Reynolds-Forchheimer pressure loss equation is

$$\frac{\Delta P}{L} = \alpha \mu \bar{U} \langle m \rangle + \beta \rho_f \bar{U}^2 \langle m \rangle^2; \quad \alpha = \frac{1}{k_D}. \quad (281)$$

By comparison with the simplified VAT (SVAT) momentum equation for constant morphological characteristics and flow field properties and only the resistance coefficient  $c_d$ ,

$$\frac{\Delta P}{L} = c_d \left( \frac{S_w}{\langle m \rangle} \right) \frac{\rho_f \bar{U}^2}{2}, \quad (282)$$

a set of transfer relationships can be found to transform Ergun-type correlations and the SVAT expression. The transfer formula (Travkin and Catton [21]) is

$$c_d = f_f = \left[ \frac{\alpha \mu}{\rho_f \bar{U}} + \beta \langle m \rangle \right] \left( \frac{2\langle m \rangle^2}{S_w} \right), \quad (283)$$

where

$$\alpha = 150 \frac{(1 - \langle m \rangle)^2}{d_p^2 \langle m \rangle^3}, \quad \beta = 1.75 \frac{(1 - \langle m \rangle)}{d_p \langle m \rangle^3}, \quad (284)$$

or

$$c_d = f_f = \frac{A}{Re_{por}} + B, \quad A = \frac{8\alpha \langle m \rangle^3}{S_w^2}, \quad B = 2\beta \frac{\langle m \rangle^3}{S_w}, \quad (285)$$

where

$$Re_{por} = \frac{4\tilde{U}\langle m \rangle}{vS_w}$$

The Ergun energy friction factor relation can be written in terms of the VAT-based formulae (Travkin and Catton [21]) as

$$\frac{\Delta p}{L} = f_{er} \left( \frac{S_w}{\langle m \rangle} \right) \frac{\rho_f \tilde{U}^2}{2} \quad (286)$$

If the Ergun correlation is written using common notation, it becomes

$$\frac{\Delta p}{L} = \left( 150 \frac{(1 - \langle m \rangle)^2}{d_p^2 \langle m \rangle^3} \right) \mu \langle m \rangle \tilde{U} + \left( 1.75 \frac{(1 - \langle m \rangle)}{d_p \langle m \rangle^3} \right) \rho_f \langle m \rangle^2 \tilde{U}^2, \quad (287)$$

and if it can be further transformed to the (SVAT) Fanning friction factor, then

$$f_{er} = \frac{A_p^*}{Re_p} + B_p^*, \quad A_p^* = \left( \frac{50(1 - \langle m \rangle)}{\langle m \rangle} \right), \quad B_p^* = \frac{3.5}{6} = 0.583, \quad (288)$$

where the particle Reynolds number is

$$Re_p = (\tilde{U} d_p) / v, \quad (289)$$

and

$$f_{er} = \frac{A_{ch}^*}{Re_{por}} + B_{ch}^*, \quad \text{with } A_{ch}^* = \frac{100}{3} = 33.33, \quad \text{and } B_{ch}^* = B_p^* = 0.583, \quad (290)$$

where

$$Re_{por} = \frac{\tilde{U} d_h}{v} = \frac{2}{3} \frac{\langle m \rangle}{(1 - \langle m \rangle)} \frac{\tilde{U} d_p}{v}, \quad \text{and } Re_{por} \left( \frac{3(1 - \langle m \rangle)}{2\langle m \rangle} \right) = Re_p = \frac{\tilde{U} d_p}{v} \quad (291)$$

The common scaling length just derived will allow a great deal of data to be brought to a common basis and allow greater confidence in predictions.

#### 4. Simulation Procedures

A large amount of data exists that demonstrates the inadequacies of the Ergun drag resistance correlation (287). This is because the Ergun correlation is used with arbitrary relationships between porosity  $\langle m \rangle$ , specific surface  $S_w$ , and pore (particle) diameter  $d_h$  when it was originally developed for granular media. How unsatisfactory it can be is shown in Fig. 5.

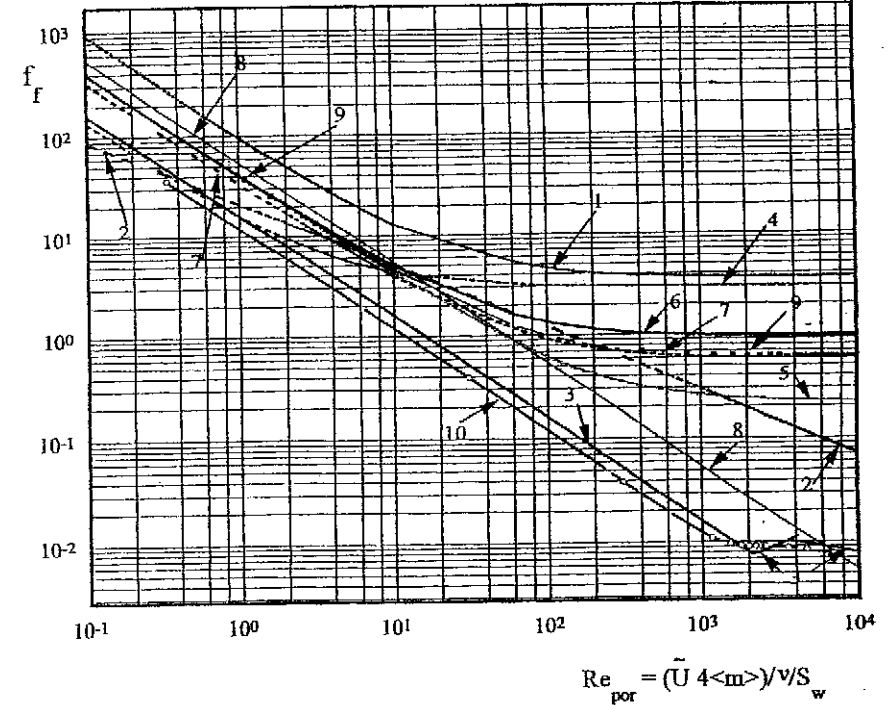


FIG. 5. Fanning friction factor  $f_f$  (bulk flow resistance in SVAT for different medium morphologies, materials, and scales used), reduced based on VAT scale transformations in experiments by 1, Gortyshov *et al.* [175]; 2, Kays and London [172]; 3, Laminar, intermediate, and turbulent laws in tube; 4, Gortyshov *et al.* [176]; 5, Beavers and Sparrow [177]; 6, SiC foam (UCLA, 1997); 7, Ergun [167]; 8, Souto and Moyne [181]; 9, Macdonald *et al.* [180]; 10, Travkin and Catton [23].

With specifically assigned morphology characteristics (primarily  $S_w$ ), the Ergun drag resistance correlation will be much closer to correlations by Beavers and Sparrow [177] and Gortyshov *et al.* [176], as shown in Fig. 5. A similar behavior was seen between the Ergun drag resistance correlation and the drag resistance correlation by Gortyshov *et al.* [175].

Several other correlations are compared in Fig. 5. Gortyshov *et al.* [175] experimentally derived correlations for the Reynolds–Forchheimer momentum equation in the form

$$\alpha = 6.61 \cdot 10^7 (\bar{d}_h)^{-1.98} \langle m \rangle^{(-4.75)}, \quad (292)$$

$$\beta = 5.16 \cdot 10^2 (\bar{d}_h)^{-1.07} \langle m \rangle^{(-11.16)}, \quad (293)$$

where hydraulic diameter  $\bar{d}_h$  (mm) is

$$\bar{d}_h = \frac{d_h [\text{m}]}{0.001 [\text{m}]} \quad (294)$$

These correlations have to be used in (285) and are for highly porous ( $\langle m \rangle = 0.87-0.97$ ) foamy metallic media. A Darcy type of friction factor obtained by Gortyshov *et al.* [176] for very low conductivity porous porcelain with high porosity is

$$f_D(Re_h) = \frac{40}{Re_h} (1 + 2.5 \cdot 10^{-2} \langle m \rangle^{-8.8} Re_h), \quad \langle m \rangle = 0.83 \div 0.92, \quad (295)$$

where

$$Re_h = \frac{\bar{U} \bar{d}_h \langle m \rangle}{\nu}$$

To transform this correlation, the Reynolds number must be transformed and the result divided by 4 to yield the Fanning friction factor,

$$f_f(Re_{por}) = \frac{1}{4} \left( \frac{40}{Re_{por} \langle m \rangle} (1 + 2.5 \cdot 10^{-2} \langle m \rangle^{-8.8} Re_{por} \langle m \rangle) \right), \quad (296)$$

with

$$Re_{por} \cong Re_h / \langle m \rangle \quad (297)$$

The correlation derived by Beavers and Sparrow [177] seems to be of little value in the original form,

$$F_{bs}(R_w) = \frac{1}{R_w} + 0.074, \quad (298)$$

because the Reynolds number,

$$R_w = \frac{\bar{U} \langle m \rangle \sqrt{k_D}}{\nu}, \quad (299)$$

contains the permeability of the medium and is usually not known. Noting that, as pointed out by Beavers and Sparrow [177] the viscous resistance coefficient  $\alpha = 1/k_D$ , where  $k_D$  is the Darcy permeability, and using the transformation

$$F_{bs} = \frac{1}{R_w} + \beta \sqrt{k_D}, \quad (300)$$

where

$$\sqrt{k_D} = \frac{1}{\sqrt{\alpha}}, \quad R_w = \frac{\bar{U} \langle m \rangle}{\nu \sqrt{\alpha}} \quad (301)$$

$$Re_{por} = R_w \frac{4}{S_w \sqrt{k_D}} = R_w \sqrt{\alpha} \left( \frac{4}{S_w} \right), \quad R_w = Re_{por} \left( \frac{S_w}{4 \sqrt{\alpha}} \right), \quad (302)$$

yields

$$F_{bs} = \frac{1}{\left( \frac{\bar{U} \langle m \rangle \sqrt{k_D}}{\nu} \right)} + \beta \sqrt{k_D}$$

or

$$F_{bs}(R_w) = \frac{1}{\sqrt{\alpha} \rho_f \bar{U}^2 \langle m \rangle^2} \left( \frac{\Delta P}{\Delta x} \right), \quad (303)$$

and when compared to

$$f_f(Re_{por}) = \frac{2 \langle m \rangle}{\rho_f \bar{U}^2 S_w} \left( \frac{\Delta P}{\Delta x} \right), \quad (304)$$

one obtains

$$\begin{aligned} f_f(R_w) &= \left( \frac{1}{\sqrt{\alpha} \langle m \rangle^2} \right) \frac{1}{\rho_f \bar{U}^2} \left( \frac{\Delta P}{\Delta x} \right) \cdot \left( \frac{\sqrt{\alpha} \langle m \rangle^2 (2 \langle m \rangle)}{S_w} \right) \\ &= F_{bs}(R_w) \left( \frac{2 \sqrt{\alpha} \langle m \rangle^3}{S_w} \right). \end{aligned} \quad (305)$$

This means that the Fanning friction factor,  $f_f$ , can be assessed from the friction factor suggested by Ward [178] and Beavers and Sparrow [177],  $f_{bs}$ , from

$$f_f(R_w) = F_{bs}(R_w) \left( \frac{2 \sqrt{\alpha} \langle m \rangle^3}{S_w} \right). \quad (306)$$

To accomplish the transformation of  $F_{bs}$  to  $f_f$ , the permeability  $k_D$  or the viscous coefficient of resistance  $\alpha$  porosity  $\langle m \rangle$  and specific surface  $S_w$  must be known. Estimates of  $f_f$  were obtained from measured values of  $F_{bs}$  for FOAMETAL (Beavers and Sparrow sample Type C) using

$$\begin{aligned} k_D &= 19.01 \cdot 10^{-4} [\text{cm}^2] = 19.01 \cdot 10^{-8} [\text{m}^2] \\ \alpha &= \frac{1}{k_D} = 0.0526 \cdot 10^8 \left[ \frac{1}{\text{m}^2} \right] \end{aligned} \quad (307)$$

and Eqs (299), (298) or (300), and (306) to transform the Beavers and Sparrow [177] experimental data correlation to the Fanning friction factor correlation. With

$$F_{bs}(R_w) = \frac{1}{R_w} + 0.074 \quad \text{and} \quad R_w = Re_{por} \left( \frac{S_w}{4\sqrt{\alpha}} \right)$$

$$F_{bs}(Re_{por}) = \frac{1}{Re_{por}} \left( \frac{4\sqrt{\alpha}}{S_w} \right) + 0.074, \quad (308)$$

then

$$f_f(Re_{por}) = \left( \frac{1}{Re_{por}} \left( \frac{4\sqrt{\alpha}}{S_w} \right) + 0.074 \right) \left( \frac{2\sqrt{\alpha}\langle m \rangle^3}{S_w} \right). \quad (309)$$

Kurshin [179] has analyzed a vast amount of data using a consistent procedure he developed to embrace all three flow regimes in porous media. To carry out the procedure, the following parameters must be known:

(a) The viscous resistant coefficient  $\alpha_1$ , evaluated for laminar flow in a pipe from the following:

$$\frac{\Delta P}{L} = \alpha_1 \mu \langle m \rangle \tilde{U}, \quad \alpha_1 = \frac{1}{k_D}, \quad \tilde{U} = \frac{d_1^2}{32\mu} \left( \frac{\Delta P}{\Delta x} \right). \quad (310)$$

(b) A characteristic length  $d_1$  evaluated by equating the preceding expressions:

$$d_1 = \left( \frac{32}{\alpha_1 \langle m \rangle} \right)^{1/2} = \left( \frac{32k_D}{\langle m \rangle} \right)^{1/2}. \quad (311)$$

(This is only justified for straight parallel capillary morphology where  $d_h = d_1$ .)

(c) Critical numbers  $Re_{er1}$  and  $Re_{er2}$  to distinguish the viscous, transitional, and turbulent filtration regimes.

(d) Dimensionless viscous  $\bar{\alpha}_3$  and inertial resistance  $\bar{\beta}_3$  coefficients in the turbulent regime. Unfortunately, Kurshin [179] did not present any data for foam materials and the porous metals he evaluated have low porosity in the range  $\langle m \rangle < 0.5$ .

Now one can say that by reformulating existing experimental correlations to the SVAT 1D form,

$$\frac{\Delta P}{L} = f_f(Re_{por}) \left( \frac{S_w}{\langle m \rangle} \right) \frac{\rho_f \tilde{U}^2}{2}, \quad (312)$$

the Fanning friction factor correlations can be easily compared with one another as they have a common consistent basis. A number of correlations were transformed and are in Fig. 5. The reason for the spread in the results is thought to be inadequate accounting for details of the medium.

Analysis of Macdonald *et al.* [180] reformulated with the help of the foregoing developed procedures gives the corrected Ergun-like type of correlation

$$f_{fM} = \frac{40}{Re_{por}} + 0.6. \quad (313)$$

Meanwhile, Souto and Moyne [181], using the DMM-DNM solutions, came to the number of resistance curves that are separate for each morphology. One of them for rectangular rods in VAT terms appears as

$$f_{fSM} = \frac{1}{3} f_{ker} = \frac{54.3}{Re_{por}}, \quad Re_{por} \rightarrow 0. \quad (314)$$

## VII. Experimental Measurements and Analysis of Internal Heat Transfer Coefficients in Porous Media

A VAT-based approach applied to heat transfer in a porous medium allows one to analyze and measure effective internal heat transfer coefficients in a porous medium. As noted by Viskanta [182], "Convective heat and mass transfer in consolidated porous materials has received practically no theoretical research attention. This is partially due to the complexity which arises as a result of physical and chemical heterogeneity that is difficult to characterize with the limited amount of data that can be obtained through experiments." Viskanta [182, 183] generalized the data he analyzed for internal heat transfer coefficient porous ceramic media using a correlation of the form

$$Nu_v = 2.0 + a Re^b Pr^{1/3}, \quad (315)$$

by assuming that the limiting Nusselt number should be 2.0 when the  $Re$  decreases to zero. This assumption is only justified for unconsolidated sparse spherical particle morphologies and is suspect for other porous medium morphologies, especially consolidated media. For this reason, some researches neglect this artificial low  $Re$  limit and correlate their findings without it. The VAT approach is applied to heat transfer in porous media to develop a more consistent correlation.

### 1. Experimental Assessment and Modeling of Heat Exchange in Porous Media

The correct form of the steady-state heat transfer equation in the fluid phase of a porous media with primarily convective 1D averaged heat transfer is

$$c_{pf}\rho_f\langle m \rangle \bar{U} \frac{\partial \bar{T}_f}{\partial x} = \frac{k_f}{\Delta\Omega} \int_{as_w} \frac{\partial T_f}{\partial x_i} \cdot \bar{d}s + k_f \frac{\partial^2 \langle m \rangle \bar{T}_f}{\partial x^2} + c_{pf}\rho_f \frac{\partial}{\partial x} \langle \langle m \rangle \{-\hat{u}\hat{T}_f\}_f \rangle + \frac{\partial}{\partial x} \left[ \frac{k_f}{\Delta\Omega} \int_{as_w} T_f \bar{d}s \right]. \quad (316)$$

Equation (316) can be rewritten as

$$\bar{\alpha}_T S_w (\{T\}_s - \{T\}_f) = - \left\{ k_f \frac{\partial}{\partial x} \left[ \frac{\partial \langle m \rangle \bar{T}_f}{\partial x} \right] + c_{pf}\rho_f \frac{\partial}{\partial x} \langle \langle m \rangle \{-\hat{u}\hat{T}_f\}_f \rangle + \frac{\partial}{\partial x} \left[ \frac{k_f}{\Delta\Omega} \int_{as_w} T_f \bar{d}s \right] \right\} + c_{pf}\rho_f \langle m \rangle \bar{U} \frac{\partial \bar{T}_f}{\partial x}, \quad (317)$$

where

$$\frac{k_f}{\Delta\Omega} \int_{as_w} \frac{\partial T_f}{\partial x_i} \cdot \bar{d}s = \bar{\alpha}_T S_w (\{T\}_s - \{T\}_f).$$

The right-hand side of Eq. (316) can also be written in the form

$$\bar{\alpha}_T S_w (\{T\}_s - \{T\}_f) + \frac{\partial}{\partial x} \left[ K_{eff,g} \frac{\partial \bar{T}_f}{\partial x} \right] = \bar{\alpha}_T S_w (\{T\}_s - \{T\}_f) + \frac{\partial}{\partial x} [-\bar{q}_{f,x}], \quad (318)$$

where the right-hand side ("diffusive"-like) flux contains more terms than are conventionally considered:

$$\bar{q}_{f,x} = \left[ -K_{eff,g} \frac{\partial \bar{T}_f}{\partial x} \right] = - \left\{ \langle m \rangle k_f \frac{\partial \bar{T}_f}{\partial x} + c_{pf}\rho_f \langle m \rangle \{-\hat{u}\hat{T}_f\}_f + \frac{k_f}{\Delta\Omega} \int_{as_w} T_f \bar{d}s \right\}. \quad (319)$$

The corresponding equation for the solid phase is

$$\frac{\partial}{\partial x} \left( \frac{\partial \langle s \rangle \{T\}_s}{\partial x} \right) + \frac{\partial}{\partial x} \left[ \frac{1}{\Delta\Omega} \int_{as_w} T_s \bar{d}s_1 \right] + \frac{1}{\Delta\Omega} \int_{as_w} \frac{\partial T_s}{\partial x_i} \cdot \bar{d}s_1 = 0. \quad (320)$$

The three terms are written in the following shorthand form:

$$T_s D_1 + T_s M D_1 + T_s M E_1 = 0. \quad (321)$$

Equation (320) can also be written

$$0 = \left( \frac{1}{k_s} \right) \bar{\alpha}_T S_w (\{T\}_f - \{T\}_s) + \frac{\partial}{\partial x} \left( k_{eff,s} \frac{\partial \{T\}_s}{\partial x} \right) = \bar{\alpha}_T S_w (\{T\}_f - \{T\}_s) + \frac{\partial}{\partial x} [-\bar{q}_{s,x}]. \quad (322)$$

Using the closure term for interface heat flux found earlier (they are equal),

$$\bar{\alpha}_T S_w (\{T\}_f - \{T\}_s) = \frac{k_s}{\Delta\Omega} \int_{as_w} \frac{\partial T_s}{\partial x_i} \cdot \bar{d}s_1.$$

Equation (322) has a term that is usually overlooked (the second term on the right):

$$\bar{q}_{s,x} = \left( -K_{eff,s} \frac{\partial \{T\}_s}{\partial x} \right) = - \left\{ \frac{\partial \langle s \rangle T_s}{\partial x} + \frac{1}{\Delta\Omega} \int_{as_w} T_s \bar{d}s_1 \right\}. \quad (323)$$

Three heat transfer coefficient models are needed to properly tie everything together. The first model incorporates only the heat transfer coefficient between the phases.

**a. Model 1 of Heat Transfer Coefficient in Porous Media: Conventional Modeling** If it is assumed that the porous medium heat transfer coefficient is defined by

$$\bar{\alpha}_{T1} = \left( \frac{k_f}{\Delta\Omega} \int_{as_w} \frac{\partial T_f}{\partial x_i} \cdot \bar{d}s \right) / [S_w (\{T\}_s - \{T\}_f)], \quad (324)$$

then the heat transfer equation becomes

$$c_{pf}\rho_f \langle m \rangle \bar{U} \frac{\partial \bar{T}_f}{\partial x} = k_f \frac{\partial}{\partial x} \left[ \frac{\partial \langle m \rangle \bar{T}_f}{\partial x} \right] + \bar{\alpha}_{T1} S_w (\{T\}_s - \{T\}_f), \quad (325)$$

and when the porosity is constant, the equation becomes

$$c_{pf}\rho_f \bar{U} \frac{\partial \bar{T}_f}{\partial x} = k_f \frac{\partial}{\partial x} \left[ \frac{\partial \bar{T}_f}{\partial x} \right] + \bar{\alpha}_{T1} S_w (\{T\}_s - \{T\}_f) / \langle m \rangle. \quad (326)$$

Most work uses an equation of this type. The experiments carried out will reflect the use of Eq. (326), and the data reduction will lead to a correlation for  $\bar{\alpha}_{T1} S_w$  that is only valid for the particular medium used in the experiment. There will be no generality in the results. By redefining  $\bar{\alpha}_{T1}$ , further medium characteristics can be incorporated into the correlation. The second model incorporates velocity and temperature fluctuations.

**b. Model 2 of Heat Transfer Coefficient in Porous Media: With Nonlinear Fluctuations** If we define the heat transfer coefficient in a way that includes the fluctuations,

$$\bar{\alpha}_{T2} = \left( \frac{k_f}{\Delta\Omega} \int_{\partial S_w} \frac{\partial T_f}{\partial x_i} \cdot \bar{d}s + c_{pf} \rho_f \frac{\partial}{\partial x} \langle \langle m \rangle \{ -\hat{u} \hat{T}_f \}_f \rangle \right) / [S_w(\{T_s\}_s - \{T_f\}_f)], \quad (327)$$

the second heat transfer model in porous media is almost the same as the first,

$$c_{pf} \rho_f \langle m \rangle \tilde{U} \frac{\partial \tilde{T}_f}{\partial x} = k_f \frac{\partial}{\partial x} \left[ \frac{\partial \langle m \rangle \tilde{T}_f}{\partial x} \right] + \bar{\alpha}_{T2} S_w(\{T_s\}_s - \{T_f\}_f). \quad (328)$$

The third model is obtained by using the complete energy equation for the fluid phase. This is again done by redefinition of the heat transfer coefficient.

**c. Model 3 of Heat Transfer Coefficient in Porous Media: Full Equation Energy Equation**

$$\bar{\alpha}_{T3} = \frac{\left( \frac{k_f}{\Delta\Omega} \int_{\partial S_w} \frac{\partial T_f}{\partial x_i} \cdot \bar{d}s + c_{pf} \rho_f \frac{\partial}{\partial x} \langle \langle m \rangle \{ -\hat{u} \hat{T}_f \}_f \rangle + \frac{\partial}{\partial x} \left[ \frac{k_f}{\Delta\Omega} \int_{\partial S_w} T_f \bar{d}s \right] \right)}{S_w(\{T\}_s - \{T\}_f)} \quad (329)$$

The energy equation is again very similar:

$$c_{pf} \rho_f \langle m \rangle \tilde{U} \frac{\partial \tilde{T}_f}{\partial x} = k_f \frac{\partial}{\partial x} \left[ \frac{\partial \langle m \rangle \tilde{T}_f}{\partial x} \right] + \bar{\alpha}_{T3} S_w(\{T_s\}_s - \{T_f\}_f). \quad (330)$$

Each of the models reflects the data obtained for a given medium. Only the coefficient  $\bar{\alpha}_{T3}$ , however, allows for a complete representation of the parameters that reflect the characteristics of the medium. In attempts by some researchers to improve the modeling, a more complete equation is used along with the more conventional definitions of the heat transfer coefficient. The relative inaccuracy of substitution of coefficient into the correct mathematical model,

$$\begin{aligned} & c_{pf} \rho_f \langle m \rangle \tilde{U} \frac{\partial \tilde{T}_f}{\partial x} + c_{pf} \rho_f \frac{\partial}{\partial x} \langle \langle m \rangle \{ \hat{u} \hat{T}_f \}_f \rangle \\ &= k_f \frac{\partial}{\partial x} \left[ \frac{\partial \langle m \rangle \tilde{T}_f}{\partial x} + \frac{\partial}{\partial x} \left[ \frac{k_f}{\Delta\Omega} \int_{\partial S_w} T_f \bar{d}s \right] \right] + \bar{\alpha}_T S_w(\{T\}_s - \{T\}_f), \quad (331) \end{aligned}$$

can easily be seen by comparison with the definition of  $\bar{\alpha}_{T3}$ . The additional terms are already a part of the coefficient, and double accounting has occurred. The seriousness of such a mistake depends on the problem.

To summarize, the heat transfer coefficients and their respectively fluid heat transport equations can be written in terms of the notation given by Eq. (321),

$$\bar{\alpha}_{T1} = (T_f M E_1) / [S_w(\{T_s\}_s - \{T_f\}_f)], \quad (332)$$

$$\left( \frac{k_f}{\Delta\Omega} \int_{\partial S_w} \frac{\partial T_f}{\partial x_i} \cdot \bar{d}s \right) / [S_w(\{T\}_s - \{T\}_f)],$$

$$\bar{\alpha}_{T2} = (T_f M E_1 + T_f M C_1) / [S_w(\{T_s\}_s - \{T_f\}_f)], \quad (333)$$

$$\left( \frac{k_f}{\Delta\Omega} \int_{\partial S_w} \frac{\partial T_f}{\partial x_i} \cdot \bar{d}s + c_{pf} \rho_f \frac{\partial}{\partial x} \langle \langle m \rangle \{ -\hat{u} \hat{T}_f \}_f \rangle \right) / [S_w(\{T_s\}_s - \{T_f\}_f)],$$

$$\bar{\alpha}_{T3} = (T_f M E_1 + T_f M C_1 + T_f M D_1) / [S_w(\{T\}_s - \{T\}_f)], \quad (334)$$

$$\frac{\left( \frac{k_f}{\Delta\Omega} \int_{\partial S_w} \frac{\partial T_f}{\partial x_i} \cdot \bar{d}s + c_{pf} \rho_f \frac{\partial}{\partial x} \langle \langle m \rangle \{ -\hat{u} \hat{T}_f \}_f \rangle + \frac{\partial}{\partial x} \left[ \frac{k_f}{\Delta\Omega} \int_{\partial S_w} T_f \bar{d}s \right] \right)}{S_w(\{T\}_s - \{T\}_f)}$$

Substitution of either of the preceding effective coefficients into the equation

$$\begin{aligned} c_{pf} \rho_f \langle m \rangle \tilde{U} \frac{\partial \tilde{T}_f}{\partial x} &= k_f \frac{\partial}{\partial x} \left[ \frac{\partial \langle m \rangle \tilde{T}_f}{\partial x} \right] + c_{pf} \rho_f \frac{\partial}{\partial x} \langle \langle m \rangle \{ -\hat{T}_f \hat{u}_i \}_f \rangle \\ &+ \frac{\partial}{\partial x} \left[ \frac{k_f}{\Delta\Omega} \int_{\partial S_w} T_f \bar{d}s \right] + \frac{1}{\Delta\Omega} \int_{\partial S_w} k_f \frac{\partial T_f}{\partial x_i} \cdot \bar{d}s, \quad (335) \end{aligned}$$

$$T_f C_1 = T_f D_1 + T_f M C_1 + T_f M C_1 + T_f M E_1,$$

would result having different models for experimental data reduction and even for experimental setup.

## 2. Simulation Procedures

Kar and Dybbs [184] developed several correlations for the internal heat transfer in different porous media. Their model for assessment of internal surface heat transfer coefficient is based on the formula (constructed slightly differently than done by Kar and Dybbs [184] but with all the features)

$$\bar{\alpha}_{T-KD} = \frac{\rho_f \tilde{U} S_{cr} (c_{p2} T_{f2} - c_{p1} T_{f1})}{S_w \Delta\Omega_f (\{T\}_s - \{T\}_f)}, \quad (336)$$

which accounts for the heat exchange when  $T_{f2}$  and  $T_{f1}$  are the temperatures of fluid exiting and entering the control volume, which is taken to be equal to  $\Delta\Omega_f$ , through cross flow surface area  $S_{cr}$  [ $m^2$ ] with mass flow rate  $M = \rho_f \bar{U} S_{cr}$  [ $kg/s$ ]. This definition of heat transfer coefficient corresponds to the continuum mathematical model of heat exchange in the porous medium formulated as

$$\langle m \rangle (\rho c_p)_f \bar{U}_i \nabla \bar{T}_f = \tilde{\alpha}_{T-KD} S_w (\{T\}_s - \bar{T}_f), \quad (337)$$

instead of the correct equation,

$$\begin{aligned} \langle m \rangle (\rho c_p)_f \bar{U}_i \nabla \bar{T}_f &= (\rho c_p)_f \nabla \cdot \langle -\bar{T}_f \hat{u}_i \rangle_f + k_f \nabla \nabla (\langle m \rangle \bar{T}_f) \\ &+ k_f \nabla \cdot \left[ \frac{1}{\Delta\Omega} \int_{\partial S_w} T_f \bar{d}s \right] + \frac{k_f}{\Delta\Omega} \int_{\partial S_w} \nabla T_f \cdot \bar{d}s. \end{aligned} \quad (338)$$

The last term can be modeled using the heat transfer coefficient given by

$$\frac{k_f}{\Delta\Omega} \int_{\partial S_w} \frac{\partial T}{\partial x_i} \cdot \bar{d}s_1 = \tilde{\alpha}_T S_w (\{T\}_2 - \{T\}_f), \quad (339)$$

which results from the closure relationship

$$\begin{aligned} \frac{1}{\Delta\Omega} \int_{\partial S_w} k_f \frac{\partial T}{\partial x_i} \cdot \bar{d}s_1 &= -\frac{1}{\Delta\Omega} \int_{\partial S_w} k_f \frac{\partial T}{\partial n_2} ds \cdot \mathbf{n}_2 \\ &= \frac{1}{\Delta\Omega} \int_{\partial S_w} \mathbf{q}_2 \cdot \bar{d}s_2 = \tilde{\alpha}_T S_w (\{T\}_2 - \{T\}_f). \end{aligned} \quad (340)$$

Kar and Dybbs measured the temperatures  $T_s$  and  $T_f$  and treated them as if they were the mean (averaged) temperatures. As a result, they measured yet another heat transfer coefficient,  $\tilde{\alpha}_{T4}$ , that is defined by

$$\begin{aligned} \tilde{\alpha}_{T4} S_w (\{T\}_s - \bar{T}_f) &= \tilde{\alpha}_{T-KD} S_w (\{T\}_s - \bar{T}_f) \\ &= (\rho c_p)_f \nabla \cdot \langle -\hat{T}_f \hat{u}_i \rangle_f + k_f \nabla \nabla (\langle m \rangle \bar{T}_f) \\ &+ k_f \nabla \cdot \left[ \frac{1}{\Delta\Omega} \int_{\partial S_w} T_f \bar{d}s \right] + \frac{k_f}{\Delta\Omega} \int_{\partial S_w} \nabla T_f \cdot \bar{d}s. \end{aligned} \quad (341)$$

The second and third terms in Eq. (341) are usually negligible. When they are, the measured heat transfer coefficient reduces to the second heat transfer coefficient in porous medium  $\tilde{\alpha}_{T2}$ ,

$$\begin{aligned} \tilde{\alpha}_{T2} S_w (\{T\}_s - \bar{T}_f) &= \tilde{\alpha}_{T-KD} S_w (\{T\}_s - \bar{T}_f) \\ &= (\rho c_p)_f \nabla \cdot \langle -\hat{T}_f \hat{u}_i \rangle_f + \frac{k_f}{\Delta\Omega} \int_{\partial S_w} \nabla T_f \cdot \bar{d}s. \end{aligned} \quad (342)$$

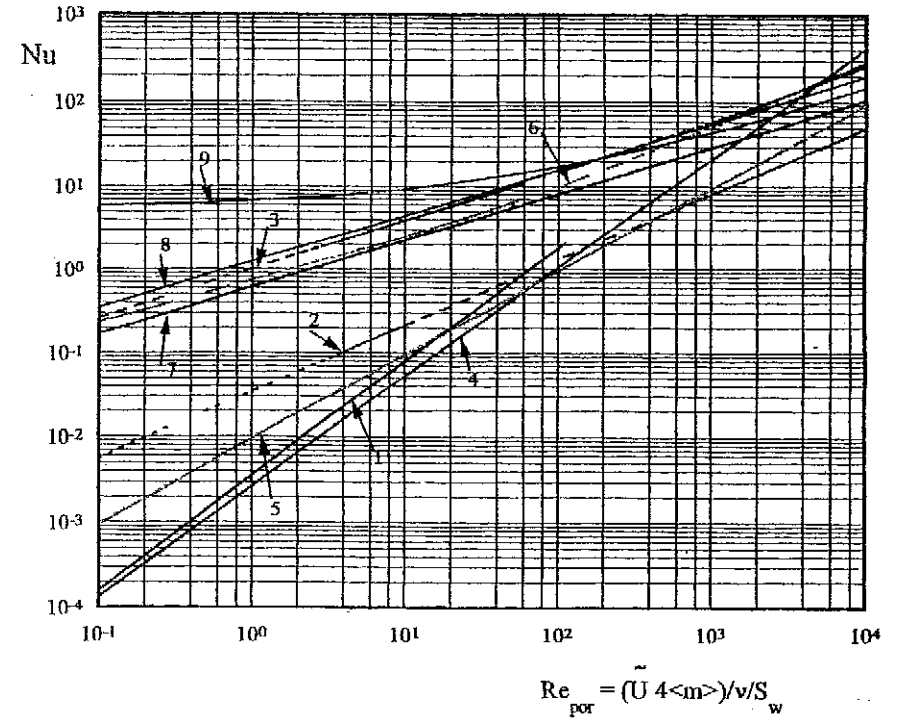


FIG. 6. Internal effective heat transfer coefficient in porous media, reduced based on VAT scale transformations in experiments by 1, Kar and Dybbs [184] for laminar regime; 2, Rajkumar [185]; 3, Achenbach [186]; 4, Younis and Viskanta [187]; 5, Galitseysky and Moshayev [189]; 6, Kokorev *et al.* [190]; 7, Gortyshov *et al.* [175]; 8, Kays and London [172]; 9, *Heat Exchangers Design Handbook* [191].

This is probably why the correlation developed by Kar and Dybbs [184] is located low among the second group of correlations in Fig. 6, where a number of correlations are presented after being rescaled using VAT. If the measured coefficient is  $\tilde{\alpha}_{T1}$ , the result will be even lower than  $\tilde{\alpha}_{T2}$ .

As the number of terms that can be estimated increases, the value of the coefficient decreases. This is probably the case with the first group of correlations shown in Fig. 6. A large amount of the data analyzed by Viskanta [182, 183] was used to deduce consistent correlations for comparison of internal porous media heat transfer characteristics. The same scaling VAT approach used for flow resistance in porous media is used for heat transfer.



One of the correlations developed by Kar and Dybbs [184], correlation (11) on p. 86, is for laminar flow in sintered powder metal specimens. It is

$$Nu_h = \frac{\bar{h}_s d_h}{\lambda_f} = 0.004 Re_h^{1.35} Pr^{1/3}, \quad (343)$$

where both  $Nu$  and  $Re$  are based on the mean pore diameter. If a single hydraulic diameter  $d_h$  is

$$d_h \cong d_{por} = \frac{4\langle m \rangle}{S_w}, \quad (344)$$

then

$$Re_h = Re_{por} = \frac{4\bar{U}\langle m \rangle}{vS_w} \quad (345)$$

$$Nu_{por}(Re_{por}) = \frac{\bar{h}_s d_{por}}{\lambda_f} \cong Nu_h(Re_h, \langle m \rangle, S_w) = 0.004 Re_{por}^{1.35} Pr^{1/3}. \quad (346)$$

This correlation is shown in Fig. 6. The correlation developed by Rajkumar [185] for hollow ceramic spheres is

$$Nu_p = \frac{\bar{h}_s d_p}{\lambda_f} = 1.1 \left( Re_p Pr \frac{d_p}{L} \right)^{0.79}, \quad (347)$$

with  $d_p = 2.5-3.5 [10^{-3}m]$ ,  $18 < Re_p < 980$ ,  $\langle m \rangle = 0.38-0.39$ ,  $Pr = 0.71$ , and

$$Re_p = \frac{\bar{u} d_p}{\nu}.$$

The particle Reynolds number  $Re_p$  can be rewritten using

$$Re_p = Re_{por} \left( \frac{3(1 - \langle m \rangle)}{2\langle m \rangle} \right). \quad (348)$$

$Nu_p$  needs to be transformed to  $Nu_{por}$  by relating the particle diameter  $d_p$  to the hydraulic diameter. The result is

$$Nu_{por} \cong \frac{\bar{h}_s d_h}{\lambda_f} = Nu_h = \frac{2\langle m \rangle}{3(1 - \langle m \rangle)} Nu_p(Re_p) = \frac{2\langle m \rangle}{3(1 - \langle m \rangle)} Nu_p(x), \quad (349)$$

where

$$x = \left( \frac{3(1 - \langle m \rangle)}{2\langle m \rangle} \right) Re_{por}.$$

Then

$$\begin{aligned} Nu_{por} &= \frac{2\langle m \rangle}{3(1 - \langle m \rangle)} Nu_p(x, Pr, d_p, L) \\ &= \frac{2\langle m \rangle}{3(1 - \langle m \rangle)} Nu_p \left( \left( \frac{3(1 - \langle m \rangle)}{2\langle m \rangle} \right) Re_{por}, Pr, d_p, L \right). \end{aligned} \quad (350)$$

Achenbach [186] developed the correlation

$$Nu_h = \left\{ (1.18 Re_h^{0.58})^4 + \left[ 0.23 \left( \frac{Re_h}{\langle m \rangle} \right)^{0.75} \right]^4 \right\}^{1/4}, \quad (351)$$

for  $Pr = 0.71$ ,  $\langle m \rangle = 0.387$ , and  $1 < (Re_h/\langle m \rangle) < 7.7 \times 10^5$ . The Reynolds number used by Achenbach is based on hydraulics and

$$Re_h \cong Re_{por} \langle m \rangle,$$

and his definition of  $Nu_h$  is

$$Nu_{por}(Re_{por}) \cong Nu_h(Re_{por} \langle m \rangle). \quad (352)$$

A correlation developed for cellular consolidated ceramics by Younis and Viskanta [187, 188] is

$$Nu_{vh} = \frac{\bar{h}_v d_h^2}{\lambda_f} = \left( 0.0098 + 0.11 \left( \frac{d_h}{L} \right) \right) Re_h^{1.3} Pr^{1/3}, \quad (353)$$

where  $\langle m \rangle = 0.83-0.87$ . The correlation yields an increasing  $Nu_{vh}$  when the test specimen thickness is decreased. This is a clear influence of inflow and outflow boundaries on heat transfer. Transforming from a volumetric Nusselt number  $Nu_v$  to a conventional surficial value  $Nu$  yields

$$Nu_{por} = \frac{Nu_{vh}(Re_{por} \langle m \rangle)}{4\langle m \rangle}. \quad (354)$$

Viskanta [183] presents a correlation from a study of low porosity media,  $0.167 < \langle m \rangle < 0.354$ , by Galitseysky and Moshaev [189]:

$$Nu_{vh} = A \langle m \rangle^{1/2} (1 - \langle m \rangle)^{3/2} Re_h Pr. \quad (355)$$

The coefficient,  $A$  given by Viskanta [183] is

$$A = \left( 37.2 \left( \frac{d_h}{L} \right) - 0.59 \right) (\langle m \rangle (1 - \langle m \rangle)^3)^{0.5}, \quad (356)$$

for  $0.15 < d_h/L < 0.23$ ,  $10 < Re_h < 530$ ,  $Pr = 0.71$ . The volumetric Nusselt number is transformed to the surficial Nusselt number with Eq. (354).

A semiempirical theory was used by Kokorev *et al.* [190] to develop a correlation between resistance coefficient and heat transfer coefficient for extensive flow regimes in porous media that only contains one empirical (apparently universal for the turbulent regime) constant. On the basis of this relationship, the concept of fluctuation speed scale of movement is used to obtain an expression for the heat transfer coefficient from the Darcy friction factor,  $f_D = 4f_f = 4c_d$ :

$$Nu_p = \frac{\bar{h}_s d_p}{\lambda_f} = [0.14(4c_d Re_p^3)^{1/4} Pr^{1/3}]. \quad (357)$$

Transforming their expression to the general form of the media Nusselt number yields

$$Nu_{por} = \frac{2\langle m \rangle}{3(1 - \langle m \rangle)} Nu_p(Re_{por} \langle m \rangle). \quad (358)$$

The heat transfer coefficient given in the *Heat Exchanger Design Handbook* [191] is based on a single sphere heat transfer coefficient for the porous medium,

$$\bar{h}_s = \frac{\lambda_f}{d_p} (f_\psi Nu_s), \quad Nu_s = 2 + (Nu_l^2 + Nu_T^2)^{1/2}, \quad (359)$$

where

$$Nu_l = 0.664 Re_p^{1/2} Pr^{1/3}$$

$$Nu_T = \frac{(0.037 Re_p^{0.8} Pr)}{(1 + 2.443 Re_p^{-0.1} (Pr^{2/3} - 1))},$$

for  $1 < Re_p < 10^5$ ,  $0.6 < Pr < 10^5$ , and the form coefficient for  $0.26 < \langle m \rangle < 1.0$  is

$$f_\psi = 1 + 1.5(1 - \langle m \rangle).$$

Transformation of the Nusselt number yields

$$Nu_{por} = \frac{2\langle m \rangle}{3(1 - \langle m \rangle)} Nu_p(Re_p). \quad (360)$$

$Nu_{por}$  values at low Reynolds number are unrealistic, leading to the conclusion that the transition type expression used to treat both laminar and turbulent flows is probably not adequate for heat transfer in porous media.

Gortyshov *et al.* [175] developed a correlation for the internal heat transfer coefficient for a highly porous metallic cellular (foamy) medium

with porosity in the range  $0.87 < \langle m \rangle < 0.97$ ,

$$Nu_{vh} = \frac{\bar{h}_v \bar{d}_h^2}{\lambda_f} = 0.606 Pe_h^{0.56} \langle m \rangle^{-5.2}, \quad (361)$$

where

$$Pe_h = Re_h Pr = \frac{\bar{U} \langle m \rangle \bar{d}_h}{a_f}, \quad (362)$$

$\bar{d}_h$  is in millimeters (see (294)), and  $Nu_{vh}$  is the volumetric internal heat transfer coefficient assessed using

$$\frac{\bar{h}_v \bar{d}_v}{\bar{h}_s \bar{d}_s} = \left( \frac{\partial S_w}{\Omega} \right)^2 = S_w^2. \quad (363)$$

Also,

$$Nu_{por}(Re_{por}) = \frac{Nu_{vh}(\langle m \rangle Re_{por})}{4\langle m \rangle}. \quad (364)$$

The correlation given by Kays and London [172] is

$$St Pr^{(2/3)} = 1.4 Re_{por}^{-0.45}, \quad (365)$$

which is transformed by

$$\frac{Nu Pr^{(2/3)}}{Re_{por} Pr} = 1.4 Re_{por}^{-0.45}, \Rightarrow Nu_{por} = 1.4 Re_{por}^{0.55} Pr^{1/3}. \quad (366)$$

Some useful observations can be made by comparing the heat transfer relationships shown in Fig. 6. One of the most significant observations is that the large differences between the correlations by Kar and Dybbs [184], Younis and Viskanta [187, 188], Rajkumar [185], and others cannot be explained if one does not take into account the specific details of the medium and the experimental data treatment. Given this, the remarkable agreement, almost coincidence, of the correlations by Kays and London [172], Achenbach [186], and Kokorev *et al.* [190] should be noted. These correlations were developed using different techniques and basic approaches. The correlation given in the *Heat Exchangers Design Handbook* [191] reflects careful adjustment in the low Reynolds number range. The correlation is not based on a specific type of medium (for example, a globular morphology with a specific globular diameter). Rather, it was developed to summarize heat transfer coefficient data in packed beds for a wide range of Reynolds numbers using an assigned globular diameter. As a result, it is not solidly based on physics, and a simple transformation from particle to pore scale does not work properly.

### VIII. Thermal Conductivity Measurement in a Two-Phase Medium

A majority of thermal conduction experiments are based on a constant heat flux through the experimental specimen and measurement of interface temperatures. Data reduction (see, for example, Uher [192]) is accomplished using

$$K = \frac{QL}{A\Delta T}, \quad (367)$$

where  $Q$  is the electrical power from heater dissipated through the specimen,  $L$  is the distance used to measure the temperature difference, and  $A$  is the uniform cross-sectional area of the sample.

#### 1. Traditional Local and Piecewise Distributed Coefficient Heat Conductivity Problem Formulations

In DMM-DNM as, for example, for a dielectric medium, the equation usually used is

$$\nabla \cdot (k(\mathbf{r})\nabla T(\mathbf{r})) = 0, \quad \mathbf{r} \in \Omega, \quad (368)$$

where the conductivity coefficient function  $k$  is

$$k(\mathbf{r}) = k_1\chi^{(1)}(\mathbf{r}) + k_2\chi^{(2)}(\mathbf{r}), \quad (369)$$

and  $\chi^{(i)}$  is the characteristic function of phase  $i = 1 \vee 2$  (see, for example, Cheng and Torquato [193]). Interface boundary conditions assumed for these equalities are

$$T_1(\mathbf{r}) = T_2(\mathbf{r}), \quad \mathbf{r} \in \partial S_{12} \quad (370)$$

$$k_1(\mathbf{n} \cdot \nabla T_1(\mathbf{r})) = k_2(\mathbf{n} \cdot \nabla T_2(\mathbf{r})), \quad \mathbf{r} \in \partial S_{12}. \quad (371)$$

#### 2. Effective Coefficients Modeling

To begin, we choose the conductivity problem and first will be treating the example of constant phase conductivity coefficient conventional equations (368) for the heterogeneous medium.

As shown elsewhere (see, for example, Travkin and Catton [21]), this mathematical statement is incorrect when the equation is applied to the volume containing both phases, even when coefficient  $k(\mathbf{r})$  is taken as a random scalar or tensorial function. The reason for this is incorrect averaging over the medium, which has discontinuities.

Conventional theories of treatment of this problem do not specify the meaning of the field  $T$ , assuming that it is the local variable, or  $-T = T(\mathbf{r})$ , where at the point  $\mathbf{r}$  the point value of potential  $T$  exists.

Next, the analysis shows that the coefficient  $k = k(\mathbf{r})$ , as long as in each separate lower scale level point  $\mathbf{r}$  there exists the local  $k$  with the value of either phase 1 or phase 2, and in each of the phases the value of  $k_i$  is constant.

In the DMM-DNM approaches the mathematical statement usually deals with the local fields, and as soon as the boundary conditions are taken in some way, the problem became formulated correctly and can be solved exactly, as in work by Cheng and Torquato [193].

Difficulties arise when the result of this solution needs to be interpreted—and this is in the majority of problem statements in heterogeneous media, in terms of nonlocal fields, but averaged in some way. The averaging procedure usually is stated as being done either by stochastic or by spatial, volumetric integration. Almost all of these averaging developments are done incorrectly because of a disregard of averaging theorems for differential operators in a heterogeneous medium. More analysis of this matter is given in work by Travkin *et al.* [115].

Further, a more complicated situation arises when the intention is to formulate and find effective transport coefficients in a heterogeneous medium. Let us consider the conductivity problem in a two-phase medium. According to most accepted mathematical statements this problem is given as (368)–(371).

#### 3. Conventional Formulation of the Effective Conductivity Problem in a Two-Phase Medium

One of the methods of closure of mathematical models of diffusion processes in a heterogeneous medium is the quasihomogeneous method (Travkin and Catton [21]). In this case, the transfer process is modeled as an ideal continuum with homogeneous effective transport characteristics instead of the real heterogeneous characteristics of a porous medium. This method of closure of the diffusive terms in the heat and mass diffusion equations results in certain limitations: (a) the two-phase medium components are without fluctuations of the type  $\hat{T}$ ,  $\hat{\varepsilon}$  in each of the phases; and (b) the transfer coefficients being constant in each of the phases (Khoroshun [194, 195]) results in reducing them to additional algebraic equations. These equations relate the unknown averaged diffusion flows in each of the phases in the form

$$\langle \vec{j} \rangle_f + \langle \vec{j} \rangle_s = -k_{eff}^* \langle \nabla T \rangle, \quad (372)$$

when for constant (effective) coefficients it is

$$-k_{eff}^f \langle \nabla T \rangle_f - k_{eff}^s \langle \nabla T \rangle_s = -k_{eff}^* \langle \nabla T \rangle, \quad (373)$$

and also

$$\langle \nabla T \rangle = \langle \nabla T \rangle_f + \langle \nabla T \rangle_s, \quad (374)$$

so it might be written as

$$(k_{eff}^f)^{-1} \langle \vec{j} \rangle_f + (k_{eff}^s)^{-1} \langle \vec{j} \rangle_s = -\langle \nabla T \rangle. \quad (375)$$

Here  $k_{eff}^f$ ,  $k_{eff}^s$  are the transfer coefficient tensors in each of the phases, and  $k_{eff}^*$  is the effective conductivity coefficient. Thus, at least in this case, the problem of closure has been reduced to finding  $k_{eff}^*$ .

Applying the closure relation, for example,

$$k_{eff}^f \langle \nabla T \rangle_f = k_{eff}^s \langle \nabla T \rangle_s, \quad (376)$$

yields the effective stagnant coefficient

$$k_{eff}^* = \frac{2k_{eff}^f k_{eff}^s}{(k_{eff}^f + k_{eff}^s)}, \quad (377)$$

which represents the lower bound of the effective stagnant conductivity for a two-phase material from the known boundaries of Hashin-Shtrikman (see, for example, [196], Kudinov and Moizhes [197]) for equal volume fraction of phases. Other closure equations for calculating the stagnant effective conductivity are found in work by Hadley [198] and by Kudinov and Moizhes [197]. The quasi homogeneous approach has several defects: (a) The basis for the quasi-homogeneous equations is in question, (b) the local fluctuation values, as well as inhomogeneity and dispersivity of the medium, are neglected, and (c) the interdependence of the correlated coefficients and arbitrary adjustment to fit data significantly reduce the generality of the results.

#### 4. VAT-Based Considerations for Heterogeneous Media Heat Conductivity Experimental Data Reduction

Let us consider the data reduction procedure of the heterogeneous material thermal conductivity experiment.

**a. Constant Heat Conductivity Coefficient** We treat the example of the constant coefficient heat transfer equation for a heterogeneous medium and show the problem in terms of conventional experimental bulk data reduction procedures and pertinent modeling equations.

Consider an experiment on determining the thermal coefficient of phase 1 (for example) in composite (or in material that is considered as being a pure substance, but really is composite) material.

The heat transport for material phase 1 is described by

$$\langle s_1 \rangle (\rho c_p)_1 \frac{\partial \bar{T}_1}{\partial t} = k_1 \nabla^2 (\langle s_1 \rangle \bar{T}_1) + k_1 \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] + \frac{k_1}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_1 \cdot \vec{d}s_1,$$

which needs the closure of the second and the third r.h.s. terms. The latter is

$$\frac{k_1}{\Delta \Omega} \int_{\partial S_{12}} \frac{\partial T}{\partial x_i} \vec{d}s_1 = \tilde{\alpha}_{21} S_{12} (\{T\}_2 - \{T\}_1), \quad (378)$$

where the closure procedure is quite applicable to description of the fluid-solid medium heat exchange and might be considered as the analogs for the case of solid-solid heat exchange, as done in many papers. The more strict and precise integration of the heat flux over the interface surface gives the exact closure for that term in governing equations for both neighboring phases.

Also considering the two terms on the r.h.s., having them as diffusion bulk terms means that

$$k_1 \nabla^2 (\langle s_1 \rangle \bar{T}_1) + k_1 \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] = \nabla \cdot [-\bar{q}_1],$$

where the "diffusive"-like flux  $\bar{q}_1$  contains some more terms than are conventionally considered,

$$\bar{q}_1 = -k_{eff,1} \nabla (\langle s_1 \rangle \bar{T}_1) = -k_1 \nabla (\langle s_1 \rangle \bar{T}_1) - \frac{k_1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1, \quad (379)$$

where the heat flux in phase 1 is determined through the averaged temperature  $\bar{T}_1$ .

So, the effective (not homogeneous) conductivity coefficient in phase 1 is

$$\begin{aligned} k_{eff,1} &= k_1 \left[ \nabla (\langle s_1 \rangle \bar{T}_1) + \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] (\nabla (\langle s_1 \rangle \bar{T}_1))^{-1} \\ &= k_1 \left[ 1 + \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 / (\nabla (\langle s_1 \rangle \bar{T}_1)) \right]. \end{aligned} \quad (380)$$

There is a difference between this introduced coefficient  $k_{eff,1}$  and that traditionally determined through the flux in phase 1, which is

$$\bar{q}_1 = [-k_{e,1} \langle \nabla T_1 \rangle_1] = -k_1 \left( \nabla (\langle s_1 \rangle \bar{T}_1) + \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right). \quad (381)$$

Arising in this situation is the effective conductivity coefficient determination

$$k_{e,1} = k_1 \left[ \nabla(\langle s_1 \rangle \tilde{T}_1) + \frac{1}{\Delta\Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] / \langle \nabla T \rangle_1$$

$$= k_1, \quad (382)$$

which is a different variable indeed and which is still the one that is not the traditional effective heterogeneous medium heat conductivity coefficient (determined in all phases),

$$\bar{q} = [-k_{eff} \langle \nabla T \rangle] = -k_{eff} [\langle \nabla T \rangle_1 + \langle \nabla T \rangle_2]$$

$$= -k_{eff} \nabla(\langle s_1 \rangle \tilde{T}_1 + \langle s_2 \rangle \tilde{T}_2) = -k_{eff} \nabla \langle T \rangle. \quad (383)$$

After those transformations the heat transfer equation in phase 1 becomes

$$\langle s_1 \rangle (\rho c_p)_1 \frac{\partial \tilde{T}_1}{\partial t} = \nabla \cdot [k_{eff,1} \nabla(\langle s_1 \rangle \tilde{T}_1)] + \tilde{\alpha}_{21} S_{12} (\{T\}_2 - \{T\}_1). \quad (384)$$

Repeating all of this for the steady-state heat conductivity equation

$$\nabla^2(\langle s_1 \rangle \tilde{T}_1) + \nabla \cdot \left[ \frac{1}{\Delta\Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] + \frac{1}{\Delta\Omega} \int_{\partial S_{12}} \nabla T_1 \cdot \vec{d}s_1 = 0, \quad (385)$$

one obtains

$$k_{eff,1} = \left[ \nabla(\langle s_1 \rangle \tilde{T}_1) + \frac{1}{\Delta\Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] (\nabla(\langle s_1 \rangle \tilde{T}_1))^{-1} \quad (386)$$

for the equation

$$\nabla \cdot [k_{eff,1} \nabla(\langle s_1 \rangle \tilde{T}_1)] + \left( \frac{\tilde{\alpha}_{21}}{k_1} \right) S_{12} (\{T\}_2 - \{T\}_1) = 0, \quad (387)$$

where  $k_{eff,1}$  does not even depend explicitly on the phase heat conductivity coefficient  $k_1$  (if the latter is taken as a constant value). Generally speaking, it depends on  $k_1$  implicitly through the boundary conditions and the conditions at the interface surface  $\partial S_{12}$ .

Of course, the situation changes if the heat exchange term (last term in (385)) is taken into account as the input correlation factor for conventional bulk effective heat conductivity coefficient  $\bar{k}_{eff,1}$  in the equation

$$\nabla \cdot [\bar{k}_{eff,1} \nabla(\langle s_1 \rangle \tilde{T}_1)] = 0. \quad (388)$$

The main reason why in the present problem treatment the interphase heat exchange term is separated from the other two terms in the r.h.s. of Eq. (385) is that this logistics gives clarity in analysis and modeling of interface

transport processes, which is not present in conventional composite medium modeling.

Also, in the more complete and challenging physics of interface transport modeling as in the third phase, this third interphase exchange term, along with the second term, is an issue tightly connected to the closure problem and to the models of interface surface transport.

**b. Nonlinear Heat Conductivity of a Pure Phase Material** Meanwhile, for materials such as high-temperature superconductors (HTSC), a constant heat conductivity coefficient is not a justifiable choice, as the usual analysis of approaches has shown above. That means complications in treating the equation with a nonlinear heat conductivity coefficient in phase 1,

$$\langle s_1 \rangle (\rho c_p)_1 \frac{\partial \{T_1\}_1}{\partial t} = \nabla \cdot [\{K_1\}_1 \nabla(\langle s_1 \rangle \{T_1\}_1)] + \nabla \cdot [\langle s_1 \rangle \{\tilde{K}_1 \nabla \tilde{T}_1\}_1]$$

$$+ \nabla \cdot \left[ \frac{\{K_1\}_1}{\Delta\Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] + \frac{1}{\Delta\Omega} \int_{\partial S_{12}} K_1 \frac{\partial T_1}{\partial x_i} \cdot \vec{d}s_1 + \langle s_1 \rangle \{S_{T_1}\}_1, \quad (389)$$

where the effective conductivity model has two additional terms, one of which reflects the mean surface temperature over the interface surface inside of the REV, and the other of which results from nonlinearity of the fields inside subvolume  $\Delta\Omega_1$ ,

$$K_{eff,1} = \left[ \{K_1\}_1 \nabla(\langle s_1 \rangle \tilde{T}_1) + \langle s_1 \rangle \{\tilde{K}_1 \nabla \tilde{T}_1\}_1 \right. \\ \left. + \frac{\{K_1\}_1}{\Delta\Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] (\nabla(\langle s_1 \rangle \tilde{T}_1))^{-1}, \quad (390)$$

which when inserted in the heat transport equation gives

$$\langle s_1 \rangle (\rho c_p)_1 \frac{\partial \tilde{T}_1}{\partial t} = \nabla \cdot [K_{eff,1} \nabla(\langle s_1 \rangle \tilde{T}_1)] + \tilde{\alpha}_{21} S_{12} (\{T\}_2 - \{T\}_1) + \langle s_1 \rangle \{S_{T_1}\}_1. \quad (391)$$

Meanwhile, when an experimentalist evaluates his or her experimental data using the equation

$$(\rho c_p)_1 \frac{\partial \tilde{T}_1}{\partial t} = \nabla \cdot [k_1 \nabla \tilde{T}_1] \quad (393)$$

with the calculation shown earlier of the thermal conductivity coefficient using experimental data, he or she makes two mistakes:

1. He is confusing the material's clear homogeneous conductivity coefficient  $k_1$  (which is the subject of his experiment) with the effective coefficient  $k_{eff,1}$  of the same phase in a composite—which is just another variable.

2. Doing data reduction as for the modeling equation

$$(\rho c_p)_1 \frac{\partial \tilde{T}_1}{\partial t} = \nabla \cdot [k_{exp,1} \nabla \tilde{T}_1] \quad (392)$$

meaning that

$$k_{eff,1} \cong k_1, \quad (394)$$

and seriously believing that he measures the real homogeneous  $k_1$  he seeks, he drops out (but in reality he takes implicitly into account) the term reflecting the exchange rate,

$$\bar{\alpha}_{21} S_{12} (\{T\}_2 - \{T\}_1), \quad (395)$$

in the composite material, which is experiencing at least two temperatures and usually a great influence of the internal exchange rate (see work by Travkin and Kushch [33, 34] and Travkin *et al.* [21]). In this way, an experimentalist makes a second mistake due to miscalculation of the influence of this additional term—yet the conductivity coefficient  $k_{exp,1}$  evaluated from experiment is not the value it is considered to be— $k_{exp,1} \neq k_1$ .

When the experimentalist's goal is the measurement, not of a bulk effective coefficient of a material, but of the pure material's conductivity coefficient, considerations regarding the issues of homogeneity and experimental data modeling are of primary interest.

The standard definition of the effective (macroscopic) conductivity tensor is determined from

$$\langle \mathbf{j} \rangle = -k_{ij}^* \langle \nabla T \rangle, \quad (396)$$

in which it is assumed that

$$\begin{aligned} \langle \mathbf{j} \rangle &= \langle \mathbf{j} \rangle_1 + \langle \mathbf{j} \rangle_2 = -k_1 \langle \nabla T \rangle_1 - k_2 \langle \nabla T \rangle_2 = -k_{ij}^* \langle \nabla T \rangle = -k_{ij}^* \nabla \langle T \rangle \\ &= -k_{ij}^* [\langle \nabla T \rangle_1 + \langle \nabla T \rangle_2] = -k_{ij}^* \langle \nabla T \rangle_1 - k_{ij}^* \langle \nabla T \rangle_2, \end{aligned} \quad (397)$$

so, for the usually assumed interface  $\partial S_{12}$  physics, the effective coefficient is determined to be

$$\begin{aligned} k_{ij}^* \langle \nabla T \rangle &= [k_1 \langle \nabla T \rangle_1 + k_2 \langle \nabla T \rangle_2] \\ &= k_1 \nabla (\langle m_1 \rangle \tilde{T}_1) + k_2 \nabla (\langle m_2 \rangle \tilde{T}_2) + (k_1 - k_2) \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \bar{d}S_1 \end{aligned} \quad (398)$$

or

$$k_{ij}^* = \left[ k_1 \nabla (\langle m_1 \rangle \tilde{T}_1) + k_2 \nabla (\langle m_2 \rangle \tilde{T}_2) + (k_1 - k_2) \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \bar{d}S_1 \right] \langle \nabla T \rangle^{-1}, \quad (399)$$

or

$$k_{ij}^* = \frac{\left[ k_1 \nabla (\langle m_1 \rangle \tilde{T}_1) + k_2 \nabla (\langle m_2 \rangle \tilde{T}_2) + (k_2 - k_1) \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_2 \bar{d}S_2 \right]}{[\langle \nabla T \rangle_1 + \langle \nabla T \rangle_2]}, \quad (400)$$

which involves knowledge of three different functions,  $\tilde{T}_1$ ,  $\tilde{T}_2$ ,  $T_{1|\partial S_{12}}$ , in the volume  $\Omega$ . This formula for the steady-state effective conductivity can be shown to be equal to the known expression

$$\begin{aligned} k_{ij}^* \langle \nabla T \rangle &= k_2 \nabla \langle T \rangle + (k_1 - k_2) \frac{1}{\Delta \Omega} \int_{\Delta \Omega_1} \nabla T d\omega \\ &= k_2 \nabla \langle T \rangle + (k_1 - k_2) \langle \nabla T \rangle_1. \end{aligned} \quad (401)$$

It is worth noting here that the known formulae for the effective heat conductivity (or dielectric permittivity) of the layered medium

$$k_e^* = \sum_{i=1} \langle m_i \rangle k_i, \quad i = 1, 2, \quad (402)$$

for a field applied parallel to the interface of layers, and

$$k_e^* = \left[ \sum_{i=1} \frac{\langle m_i \rangle}{k_i} \right]^{-1} \quad (403)$$

when the heat flux is perpendicular to the interface, are easily derived from the general expression (399) using assumptions that intraphase fields are equal,  $\tilde{T}_1 = \tilde{T}_2$ , that interface boundary conditions are valid for averaged fields, and that adjoining surface interface temperatures are close in magnitude. The same assumptions are effectual when conventional volume averaging techniques are applied toward the derivation of formulae (402) and (403).

### 5. Bulk Heat Conductivity Coefficients of a Composite Material

The problem becomes no easier in the case when the effective conductivity coefficient is meant to serve for the whole composite material. Combining

both temperature equations (if only two phases are present) for the simplest case of constant coefficients,

$$\begin{aligned} \langle s_1 \rangle (\rho c_p)_1 \frac{\partial \tilde{T}_1}{\partial t} &= k_1 \nabla^2 (\langle s_1 \rangle \tilde{T}_1) + k_1 \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] + \frac{k_1}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_1 \cdot \vec{d}s_1 \\ \langle s_2 \rangle (\rho c_p)_2 \frac{\partial \tilde{T}_2}{\partial t} &= k_2 \nabla^2 (\langle s_2 \rangle \tilde{T}_2) + k_2 \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_2 \vec{d}s_2 \right] + \frac{k_2}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_2 \cdot \vec{d}s_2, \end{aligned}$$

into one equation by adding one to another, we obtain

$$\begin{aligned} \langle s_1 \rangle (\rho c_p)_1 \frac{\partial \tilde{T}_1}{\partial t} + \langle s_2 \rangle (\rho c_p)_2 \frac{\partial \tilde{T}_2}{\partial t} &= \nabla \cdot (k_1 \nabla (\langle s_1 \rangle \tilde{T}_1) + k_2 \nabla (\langle s_2 \rangle \tilde{T}_2)) \\ &+ \nabla \cdot \left[ \frac{k_1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 + \frac{k_2}{\Delta \Omega} \int_{\partial S_{12}} T_2 \vec{d}s_2 \right] \\ &+ \frac{k_1}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_1 \cdot \vec{d}s_1 + \frac{k_2}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_2 \cdot \vec{d}s_2, \end{aligned} \quad (404)$$

keeping in mind that the two-phase averaged temperature is

$$\langle T \rangle = \langle s_1 \rangle \tilde{T}_1 + \langle s_2 \rangle \tilde{T}_2. \quad (405)$$

One can write down the mixture temperature equation when summation of the equations gives (when taking into account the boundary condition of temperature fluxes equality at the interface surface,  $(k_1 \nabla T_1) = (k_2 \nabla T_2)$ )

$$\begin{aligned} \langle s_1 \rangle (\rho c_p)_1 \frac{\partial \tilde{T}_1}{\partial t} + \langle s_2 \rangle (\rho c_p)_2 \frac{\partial \tilde{T}_2}{\partial t} \\ = \nabla \cdot (k_1 \nabla (\langle s_1 \rangle \tilde{T}_1) + k_2 \nabla (\langle s_2 \rangle \tilde{T}_2)) + (k_1 - k_2) \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right], \end{aligned} \quad (406)$$

or, written in terms of thermal diffusivities  $a_1$  and  $a_2$ ,

$$\begin{aligned} \frac{\partial \langle T \rangle}{\partial t} &= \nabla \cdot [a_1 \nabla (\langle s_1 \rangle \tilde{T}_1) + a_2 \nabla (\langle s_2 \rangle \tilde{T}_2)] + (a_1 - a_2) \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] \\ &+ a_1 \left( 1 - \frac{a_2 k_1}{a_1 k_2} \right) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_1 \cdot \vec{d}s_1 \right], \quad a_i = \frac{k_i}{(\rho c_p)_i}, \quad i = 1, 2, \end{aligned} \quad (407)$$

which has the three different temperatures  $-\tilde{T}_1$ ,  $\tilde{T}_2$ , and  $T_1(\partial S_{12})$  (here  $\langle T \rangle = \langle s_1 \rangle \tilde{T}_1 + \langle s_2 \rangle \tilde{T}_2$ ).

And, assuming only a local thermal equilibrium,

$$\langle T \rangle = \langle s_1 \rangle \tilde{T}_1 + \langle s_2 \rangle \tilde{T}_2 = T^* = \tilde{T}_1 = \tilde{T}_2, \quad (408)$$

the mixed temperature equation becomes two-temperature  $T^*$ ,  $T_1(\partial S_{12})$  dependable with simplified left hand part of the equation

$$\begin{aligned} \langle s_1 \rangle (\rho c_p)_1 + \langle s_2 \rangle (\rho c_p)_2 \frac{\partial T^*}{\partial t} &= \nabla \cdot [(k_1 \nabla (\langle s_1 \rangle T^*) + k_2 \nabla (\langle s_2 \rangle T^*)) \\ &+ (k_1 - k_2) \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right]]. \end{aligned} \quad (409)$$

With the two different temperatures, the effective coefficient of conductivity is equal to

$$\begin{aligned} k_{eff}^* &= \left\{ [(k_1 \nabla (\langle s_1 \rangle T^*) + k_2 \nabla (\langle s_2 \rangle T^*)) \right. \\ &\left. + (k_1 - k_2) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right]] \right\} (\langle \nabla T^* \rangle)^{-1}. \end{aligned} \quad (410)$$

This formula coincides with the effective coefficient of conductivity for the steady-state effective conductivity in the medium and can be shown to be equal to the known expression

$$k_{eff}^* \nabla \langle T \rangle = k_2 \nabla \langle T \rangle + (k_1 - k_2) \frac{1}{\Delta \Omega} \int_{\Delta \Omega_1} \nabla T d\omega. \quad (411)$$

From this formula an important conclusion can be drawn: that the most sought-after characteristics in heterogeneous media transport, which are the effective transport coefficients, can be correctly determined using the conventional definition for the effective conductivity—for example, for the steady-state problem

$$-\langle j \rangle = k_{eff}^* \nabla \langle T \rangle = k_2 \nabla \langle T \rangle + (k_1 - k_2) \frac{1}{\Delta \Omega} \int_{\Delta \Omega_1} \nabla T d\omega, \quad (412)$$

but only in a fraction of problems, while employing the DMM-DNM exact solution. The issue is that in a majority of problems, such as for inhomogeneous, nonlinear coefficients and in many transient problems, having the two-field DMM-DNM exact solution is not enough to find effective coefficients. As shown earlier, only the requirement of thermal equilibrium warrants the equality of steady-state and transient effective conductivities in a two-phase medium.

The second form of the same equation with the surface integral of the fluctuation temperature in phase 1 is

$$\begin{aligned} \langle s_1 \rangle (\rho c_p)_1 + \langle s_2 \rangle (\rho c_p)_2 \frac{\partial T^*}{\partial t} = \nabla \cdot [(k_1 \langle s_1 \rangle + k_2 \langle s_2 \rangle) \nabla(T^*)] \\ + (k_1 - k_2) \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} \hat{T}_1 \vec{d}s_1 \right], \end{aligned} \quad (413)$$

still having the phase 1 temperature fluctuation variable in one of the terms. The following equality arises while comparing the two last equations (409) and (413):

$$\begin{aligned} [(k_1 \nabla(\langle s_1 \rangle T^*) + k_2 \nabla(\langle s_2 \rangle T^*)) + (k_1 - k_2) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right]] \\ = [(k_1 \langle s_1 \rangle + k_2 \langle s_2 \rangle) \nabla(T^*) + (k_1 - k_2) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} \hat{T}_1 \vec{d}s_1 \right]]. \end{aligned} \quad (414)$$

As can be seen, the transient effective diffusivity coefficient  $a_{eff}^o$  in the VAT nonequilibrium two-temperature equation (407) can be derived through the equality

$$\begin{aligned} a_{eff}^o \langle \nabla T \rangle = a_1 \nabla(\langle s_1 \rangle \tilde{T}_1) + a_2 \nabla(\langle s_2 \rangle \tilde{T}_2) + (a_1 - a_2) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] \\ + \nabla^{(-1)} \left\{ a_1 \left( 1 - \frac{a_2 k_1}{a_1 k_2} \right) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_1 \cdot \vec{d}s_1 \right] \right\} \end{aligned} \quad (415)$$

or

$$a_{eff}^o \langle \nabla T \rangle = a_1 \nabla(\langle s_1 \rangle \tilde{T}_1) + a_2 \nabla(\langle s_2 \rangle \tilde{T}_2) + (a_1 - a_2) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_1 \vec{d}s_1 \right] + \Lambda, \quad (416)$$

where  $\nabla^{(-1)}$  is the inverse operator  $-\nabla \cdot (\nabla^{(-1)}(f)) = f$  such that if

$$\nabla \cdot \mathbf{A} = a_1 \left( 1 - \frac{a_2 k_1}{a_1 k_2} \right) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_1 \cdot \vec{d}s_1 \right], \quad (417)$$

then

$$\mathbf{A} = \nabla^{(-1)} \left\{ a_1 \left( 1 - \frac{a_2 k_1}{a_1 k_2} \right) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_1 \cdot \vec{d}s_1 \right] \right\}. \quad (418)$$

From the preceding expression, the transient effective nonequilibrium coefficient in a two-phase medium can be determined as

$$k_{eff}^o = a_{eff}^o (\langle s_1 \rangle (\rho c_p)_1 + \langle s_2 \rangle (\rho c_p)_2), \quad (419)$$

which looks rather inconvenient for analytical or experimental assessment or numerical calculation. The solution of this problem, which includes as an imperative part the finding of the effective bulk composite material heat conductivity (diffusivity), coefficient, is equal to the solution of the exact two-phase problem. We see that the two-temperature DMM-DNM is not enough for the convenient construction of the effective coefficient of conductivity. As we can compare the expressions for transient coefficient (419) and thermal equilibrium coefficient (410) they are of great difference in definition and in calculation. And it does not matter which kind of mathematical statement is used for the problem—the two separate heat transfer equations or the VAT statement—the problem complexity is the same. Only by using the VAT equations is the correct estimation of the transient effective coefficients on the upper scale available.

If we adopt the idea that phase temperature variables in each of the subvolumes  $\Delta \Omega_1$  and  $\Delta \Omega_2$  can be presented as sums of the overall temperature and local fluctuations (Nozad *et al.* [40]),

$$\tilde{T}_1 = \langle T \rangle + \check{T}_1, \quad \tilde{T}_2 = \langle T \rangle + \check{T}_2, \quad (420)$$

which means an introduction of the two new variables  $\check{T}_1$  and  $\check{T}_2$ , then the equation for the composite averaged temperature follows (Nozad *et al.* [40]) in the form

$$\begin{aligned} \langle s_1 \rangle (\rho c_p)_1 + \langle s_2 \rangle (\rho c_p)_2 \frac{\partial \langle T \rangle}{\partial t} = \nabla \cdot \left\{ \langle s_1 \rangle k_1 \left[ \nabla \langle T \rangle + \frac{1}{\Delta \Omega_1} \int_{\partial S_{12}} \hat{T}_1 \vec{d}s_1 \right] \right. \\ \left. + \langle s_2 \rangle k_2 \left[ \nabla \langle T \rangle + \frac{1}{\Delta \Omega_2} \int_{\partial S_{12}} \hat{T}_2 \vec{d}s_2 \right] \right\} \\ - \left\{ \langle s_1 \rangle (\rho c_p)_1 \frac{\partial \check{T}_1}{\partial t} + \langle s_2 \rangle (\rho c_p)_2 \frac{\partial \check{T}_2}{\partial t} \right. \\ \left. - \nabla \cdot (\langle s_1 \rangle k_1 \nabla \check{T}_1 + \langle s_2 \rangle k_2 \nabla \check{T}_2) \right\} \end{aligned} \quad (421)$$

which has five variable temperatures. If the assumptions and constraints given in Nozad *et al.* [40] are all satisfied, then the final equation with only



three different temperatures resumes:

$$\begin{aligned} \langle s_1 \rangle \langle \rho c_p \rangle_1 + \langle s_2 \rangle \langle \rho c_p \rangle_2 \frac{\partial \langle T \rangle}{\partial t} = \nabla \cdot \left\{ \langle s_1 \rangle k_1 \left[ \nabla \langle T \rangle + \frac{1}{\Delta \Omega_1} \int_{\partial S_{12}} \hat{T}_1 \vec{d}s_1 \right] \right. \\ \left. + \langle s_2 \rangle k_2 \left[ \nabla \langle T \rangle + \frac{1}{\Delta \Omega_2} \int_{\partial S_{12}} \hat{T}_2 \vec{d}s_2 \right] \right\}. \end{aligned} \quad (422)$$

This means that the neglect of the global deviation  $\hat{T}_1$ ,  $\hat{T}_2$  terms still does not remove the requirement of a two-temperature solution.

**a. Effective Conductivity Coefficients in a Porous Medium When Phase One Is a Fluid** In phase 1 the VAT equation is written for the laminar regime. In the work by Kuwahara and Nakayama [199] is given the DMM-DNM solution of the 2D problem of uniformly located quadratic rods with equal spacing in both directions. Studies were undertaken of both the Forchheimer and post-Forchheimer flow regimes.

This work is a good example of how DMM-DNM goals cannot be accomplished, even if the solution on the microlevel is obtained completely, if the proper VAT scaling procedures basics are not applied.

The one structural unit—periodic cell in the medium—was taken for DMM-DNM.

Equations were taken with constant coefficients, and in phase 1 the VAT equation was written for the laminar regime as

$$\begin{aligned} \langle m \rangle \langle \rho c_p \rangle_f \frac{\partial \tilde{T}_f}{\partial t} + \langle m \rangle \langle \rho c_p \rangle_f \tilde{U}_i \nabla \tilde{T}_f = \langle \rho c_p \rangle_f \nabla \cdot \langle -\hat{T}_f \hat{u}_i \rangle_f + k_f \nabla \nabla \langle \langle m \rangle \tilde{T}_f \rangle \\ + k_f \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_w} T_f \vec{d}s \right] + \frac{k_f}{\Delta \Omega} \int_{\partial S_w} \nabla T_f \cdot \vec{d}s. \end{aligned} \quad (423)$$

Adding this equation to the VAT solid-phase (second phase) two-temperature equation gives

$$\begin{aligned} \langle m \rangle \langle \rho c_p \rangle_f \frac{\partial \tilde{T}_f}{\partial t} + \langle s_2 \rangle \langle \rho c_p \rangle_2 \frac{\partial \tilde{T}_2}{\partial t} + \langle m \rangle \langle \rho c_p \rangle_f \tilde{U}_i \nabla \tilde{T}_f \\ = \langle \rho c_p \rangle_f \nabla \cdot \langle -\hat{T}_f \hat{u}_i \rangle_f + \nabla \cdot (k_f \nabla \langle \langle m \rangle \tilde{T}_f \rangle + k_2 \nabla \langle \langle s_2 \rangle \tilde{T}_2 \rangle) \\ + \nabla \cdot \left[ \frac{k_f}{\Delta \Omega} \int_{\partial S_{12}} T_f \vec{d}s_1 + \frac{k_2}{\Delta \Omega} \int_{\partial S_{12}} T_2 \vec{d}s_2 \right] \\ + \frac{k_f}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_f \cdot \vec{d}s_1 + \frac{k_2}{\Delta \Omega} \int_{\partial S_{12}} \nabla T_2 \cdot \vec{d}s_2, \end{aligned} \quad (424)$$

which reduces because of interface flux equality to

$$\begin{aligned} \langle m \rangle \langle \rho c_p \rangle_f \frac{\partial \tilde{T}_f}{\partial t} + \langle s_2 \rangle \langle \rho c_p \rangle_2 \frac{\partial \tilde{T}_2}{\partial t} + \langle m \rangle \langle \rho c_p \rangle_f \tilde{U}_i \nabla \tilde{T}_f \\ = \nabla \cdot (k_f \nabla \langle \langle m \rangle \tilde{T}_f \rangle + k_2 \nabla \langle \langle s_2 \rangle \tilde{T}_2 \rangle) + \langle \rho c_p \rangle_f \nabla \cdot \langle -\hat{T}_f \hat{u}_i \rangle_f \\ + (k_f - k_2) \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_f \vec{d}s_1 \right], \end{aligned} \quad (425)$$

which has two averaged temperatures  $\tilde{T}_f$  and  $\tilde{T}_2$ , interface surface integrated temperature  $T_f(\partial S_{12})$ , and two fields of fluctuations  $\hat{T}_f(\mathbf{x})$  and  $\hat{u}_i(\mathbf{x})$ , assuming that the velocity field is also computed and known.

We now write the effective conductivity coefficients for (425) and for the one-temperature equation when temperature equilibrium is assumed.

In the first case, for the weighted temperature,

$$\langle T^w \rangle = (\langle m \rangle \langle \rho c_p \rangle_f \tilde{T}_f + \langle s_2 \rangle \langle \rho c_p \rangle_2 \tilde{T}_2) / w_T \quad (426)$$

$$w_T = \langle m \rangle \langle \rho c_p \rangle_f + \langle s_2 \rangle \langle \rho c_p \rangle_2 = \text{const}, \quad (427)$$

the equation can be written as

$$\begin{aligned} w_T \frac{\partial \langle T^w \rangle}{\partial t} + \langle m \rangle \langle \rho c_p \rangle_f \tilde{U}_i \nabla \tilde{T}_f \\ = \nabla \cdot (k_f \nabla \langle \langle m \rangle \tilde{T}_f \rangle + k_2 \nabla \langle \langle s_2 \rangle \tilde{T}_2 \rangle) + \langle \rho c_p \rangle_f \nabla \cdot \langle -\hat{T}_f \hat{u}_i \rangle_f \\ + (k_f - k_2) \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_f \vec{d}s_1 \right], \end{aligned} \quad (428)$$

where three temperatures are unknown,  $\langle T^w \rangle$ ,  $\tilde{T}_f$ , and  $\tilde{T}_2$ , plus the interface surface temperature integral  $T_f(\partial S_{12})$  and fluctuation fields  $\hat{T}_f(\mathbf{x})$  and  $\hat{u}_i(\mathbf{x})$ .

The effective coefficient of conductivity can be looked for is

$$\begin{aligned} k_{eff}^o \langle \nabla T^w \rangle = (k_f \nabla \langle \langle m \rangle \tilde{T}_f \rangle + k_2 \nabla \langle \langle s_2 \rangle \tilde{T}_2 \rangle) + \langle \rho c_p \rangle_f \langle -\hat{T}_f \hat{u}_i \rangle_f \\ + (k_f - k_2) \left[ \frac{1}{\Delta \Omega} \int_{\partial S_{12}} T_f \vec{d}s_1 \right]. \end{aligned} \quad (429)$$

In order to avoid the complicated problems with effective conductivity coefficient definition in a multitemperature environment, Kuwahara and Kakayama [199], while performing DMM-DNM for the problem of laminar regime transport in a porous medium, decided to justify the local thermal equilibrium condition

$$\langle T \rangle = \langle m \rangle \tilde{T}_f + \langle s_2 \rangle \tilde{T}_2 = T^* = \tilde{T}_f = \tilde{T}_2,$$

which greatly changes the one effective temperature equation. This equation

becomes simpler with only one unknown temperature  $T^*$  and variable field  $\hat{T}_f$  and is written as

$$\begin{aligned} & \langle m \rangle (\rho c_p)_f + \langle s_2 \rangle (\rho c_p)_2 \frac{\partial T^*}{\partial t} + \langle m \rangle (\rho c_p)_f \tilde{U}_i \nabla T^* \\ & = \nabla \cdot (k_f \nabla \langle m \rangle T^*) + k_2 \nabla \langle s_2 \rangle T^* + (\rho c_p)_f \nabla \langle -\hat{T}_f \hat{u}_i \rangle_f \\ & + (k_f - k_2) \nabla \cdot \left[ \frac{1}{\Delta \Omega} \int_{\partial s_{12}} T_f \vec{d}s_1 \right], \end{aligned} \quad (430)$$

as the variable temperature and velocity fluctuation fields  $\hat{T}_f$  and  $\hat{u}_i$  should be known, although this is a problem. As long as the definition of the effective conductivity coefficient is

$$\begin{aligned} k_{eff}^* \langle \nabla T^* \rangle & = k_f \nabla \langle m \rangle T^* + k_2 \nabla \langle s_2 \rangle T^* + (\rho c_p)_f \langle -\hat{T}_f \hat{u}_i \rangle_f \\ & + (k_f - k_2) \left[ \frac{1}{\Delta \Omega} \int_{\partial s_{12}} T_f \vec{d}s_1 \right], \end{aligned} \quad (431)$$

then the effective conductivity can be calculated subject to known  $T^*$ ,  $\hat{T}_f$ ,  $T_f$ , and  $\hat{u}_i$ . At the same time, the important issue is that in DMM-DNM the assumption of thermal equilibrium has no sense at all—as long as the problem have been already calculated as the two-temperature problem.

To further perform the correct estimation or calculation of effective characteristics, one needs to know what are those characteristics in terms of definition and mathematical description or model?

This is the one more place where the DMM-DNM as it is performed now is in trouble if it does not comply with the same hierarchical theory derivations and conclusions as the VAT (see also the studies by Travkin *et al.* [115] and Travkin and Catton [114, 21]).

As shown earlier, only the requirement of thermal equilibrium warrants the equality of steady-state and transient effective conductivities in a two-phase medium.

Consequently, if taken correctly, the two-temperature model will introduce more trouble in treatment and even interpretation of the needed bulk, averaged temperature (as long as this problem is already known to exist and is treated in nonlinear and temperature-dependent situations) and the corresponding effective conductivity coefficient (or coefficients).

1. Thus, comparing the two effective conductivity coefficients (429) and (431), one can assess the difference in the second term form and consequently, the value of computed coefficients.

Comparing the expressions for one equilibrium temperature and one effective weighted temperature, as well as for their effective conductivity coefficients, one can also observe the great imbalance and inequality in their definitions and computations.

2. Summarizing application of DMM-DNM approach by Kuwahara and Nakayama [199], it can be said that it is questionable procedure to make an assumption of equilibrium temperatures when the problem was stated and computed as via DNM for two temperatures.
3. In the calculation of the effective coefficients of conductivity—stagnant thermal conductivity  $k_e$ ; tortuosity molecular diffusion  $k_{tor}$ ; and thermal dispersion  $k_{dis}$ —Kuwahara and Nakayama [199] used a questionable procedure for calculation of the two last coefficients.

They used one-cell (REV) computation for surface and fluctuation temperatures for periodical morphology of the medium, and at the same time they used the infinite REV definition for the effective temperature gradient for their calculation (assigned in the problem); see the expressions for calculation of these coefficients, (21)–(24) on p. 413. That action means the mixture of two different scale variables in one expression for effective characteristics—which is incorrect by definition. If this is used consciously, the fact should be stated on that matter explicitly, because it alters the results.

## IX. VAT-Based Compact Heat Exchanger Design and Optimization

At the present time, compact heat exchanger (CHE) design is based primarily on utilization of known standard heat exchanger calculation procedures (see, for example, Kays and London [172]). Typical analysis of a heat exchanger design depends on the simple heat balance equations that are widely used in the process equipment industry. Analytically based models are composed for a properly constructed set of formulas for a given spatial design of heat transfer elements that allow, most of the existing heat transfer mechanisms to be accounted for.

Analogies between heat transfer and friction have been shown by Churchill [200] and by Churchill and Chan [201] to be inadequate for describing many of the HE configurations of interest. This has been suspected for some time and will seriously affect the use of the “j-factor” in HE modeling and design.

Modeling of a specific heat exchanger geometry by Tsay and Weinbaum [202] provides a useful preliminary step and a potential benchmark test case. Though the study only considered hydrodynamic effects and restricted itself to consideration of regular media and the creeping flow regime, the effects of morphology-characteristic variation upon momentum transport phenomena were explored. The authors show that the overall bed drag coefficient in the creep flow regime increases dramatically as the inner-cylinder spacing approaches the order of the channel half-height.

Analysis of processes in regular and randomly organized heterogeneous media and CHE can be performed in different ways. Some CHE structures have the characteristics of a porous medium and can be studied by application of the developments of porous media modeling. In this work, a theoretical basis for employing heat and momentum transport equations obtained from volume averaging theory (VAT) is developed for modeling and design of heat exchangers. Using different flow regime transport models, equation sets are obtained for momentum transport and two- and three-temperature transfer in nonisotropic heterogeneous CHE media with accounting for interphase exchange and microroughness.

The development of new optimization problems based on the VAT-formulated CHE models using a dual optimization approach is suggested. Dual optimization is the optimization of the morphological parameters (size, morphology of working spaces) and the thermophysical properties (characteristics) of the working solid and liquid materials to maximize heat transfer while minimizing pressure loss. This allows heat exchanger modeling and possible optimization to be based on theoretically correct field equations rather than the usual balance equations. The problems of shape optimization traditionally have been addressed in HE design on the basis of general statements that include heat and momentum equations along with their boundary conditions stated on the assigned known volumes and surfaces; see, for example, Bejan and Morega [203].

#### A. A SHORT REVIEW OF CURRENT PRACTICE IN HEAT EXCHANGER MODELING

Analysis of heat exchanger designs, as described by Butterworth [204], depends on the heat balance equations that are widely used in the heat design industry. The general form of the thermal design equation for heat exchangers (see, for example, Figs. 7–9) can be written (Butterworth [204])

$$dQ = \alpha dA \Delta T,$$

where  $Q$  is the heat rate, and  $A$  is the transfer surface area. As outlined by Martin [205], the coupled differential equations for a cross flow tube heat exchanger (Fig. 7) modeling are (for simplicity only one row is considered)

$$\begin{aligned} -\frac{d\Theta_1}{d\zeta_1} &= \Theta_1 - \Theta_2^{av} \\ -\frac{d\Theta_2}{d\zeta_2} &= \Theta_1 - \Theta_2, \end{aligned}$$

where  $\Theta_1$ ,  $\Theta_2$ , and  $\Theta_2^{av}$  are dimensionless first and second fluid temperatures

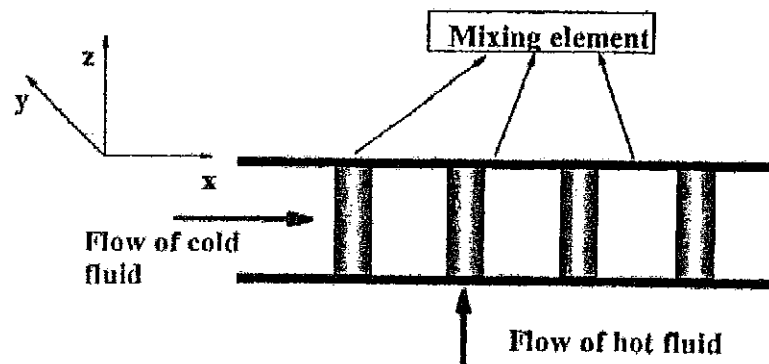


FIG. 7. Three-phase tube heat exchanger unconsolidated morphology.

and the second temperature being averaged over the tube's row width. As follows from these equations, all information about a given heat exchanger's peculiarities and design specifics is included in the dimensionless coordinates

$$\zeta_1 = \frac{\alpha A}{(Mc_p)_i} \frac{z_i}{L_i}, \quad i = 1, 2,$$

where  $\alpha$  is the overall heat transfer coefficient and  $M$  is the mass flow rate. Second-order ordinary differential equations are developed for HE as well (see, for example, Paffenbarger [206]).

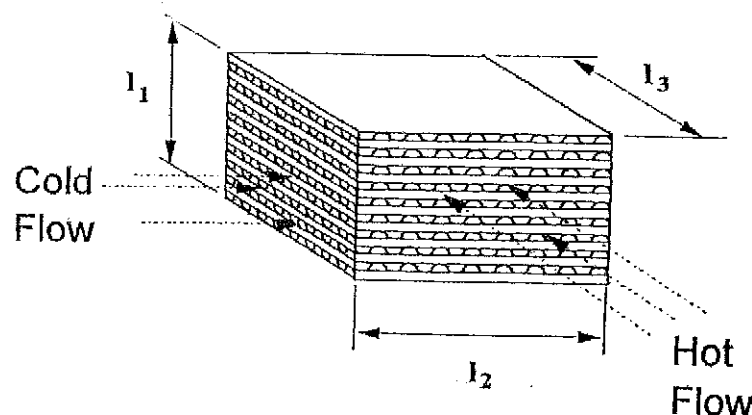


FIG. 8. CHE morphology with separated subchannels for each of the fluids.

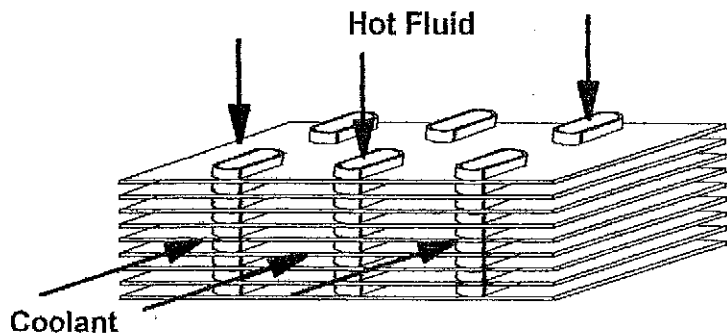


FIG. 9. Compact heat exchanger (CHE) with contracted-tube layer morphology for one of the fluids.

Webb in a book [207], and in his invited talk at the 10th International Heat Transfer Conference [208], distinguishes four basic approaches to predicting the heat transfer  $j$ -factor and the Fanning friction factor  $f$  for heat exchanger design. They are (1) power-law correlations; (2) asymptotic correlations; (3) analytically based models; and (4) numerical solutions.

Analytically based models are properly constructed set of formulas for a given spatial construction of heat transfer elements that allows most of the existing heat transfer mechanisms to be accounted for. Many examples are given in publications by Webb [207, 208], Bergles [209], and other researchers.

The major differences between the measured characteristics of air-cooled heat exchangers with aluminum or copper finned tubes with large height, small thickness, and narrow-pitch fins, and high-temperature waste heat recovery exchangers with steel finned tubes with rather low height and thickness and wide-pitched fins, are given in a paper by Fukagawa *et al.* [210]. Despite the fact that morphology of the heat exchange medium is essentially the same, the correlations predicting heat transfer and pressure drop values do not work for both HE types altogether. For this particular heat exchange morphology, a wide-ranging experiment program is needed for different ratios of the morphology parameters. There is, at present, no general approach for describing the dependencies of heat transfer effectiveness or frictional losses for a reasonably wide range of morphological properties and their ratios.

The field of compact heat exchangers has received special attention during the past several years. A wide variety of plate fin heat exchangers (PFHE) has been developed for applications in heat recovery systems, seawater evaporators, condensers for heat pumps, etc. It is proposed that a theoretical

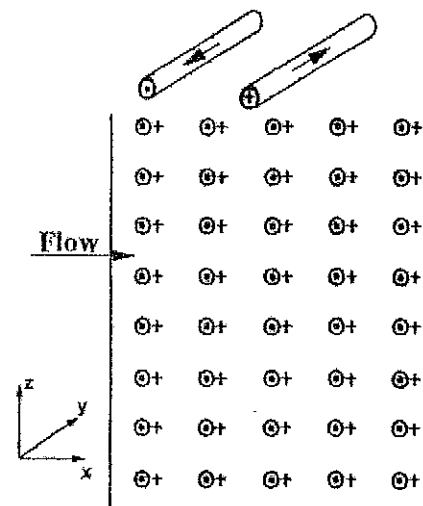


FIG. 10. Initial optimization scheme for benchmark tube heat exchanger morphology.

basis for employing heat and momentum transport equations obtained with volume averaging theory be developed for the design of heat exchangers.

An assumption of the equilibrium streams is common in HE design (see, for example, Butterworth [204]). Almost all commercial design software assume plug flow with occasional simple corrections to reflect deviations from the plug flow. CFD has applications in simplified situations, when the geometry of the channels or heat transfer surfaces can be described fairly. Butterworth [204] further noted that “the space outside tubes in heat exchangers presents an enormously complicated geometry” and “modeling these exchangers fully, even with simplified turbulence models just mentioned is still impracticable.” We do not agree with this view and propose to use techniques developed as part of our work to show that practical modeling methods exist.

During the past few years considerable attention has been given to the problem of active control of fluid flows. This interest is motivated by a number of potential applications in areas such as control of flow separation, combustion, fluid–structure interaction, and supermaneuverable aircraft. In this direction, Burns *et al.* [211, 212] developed several computational algorithms for active control design for the Burgers equation, a simple model for convection–diffusion phenomena such as shock waves and traffic flows.

Generally, the optimal control problems with partial differential equations (PDE), to which VAT-based HE models convert, can have detailed