Buyevich's works

Buyevich, Y.A. and Theofanous, T.G., (1997), "Ensemble Averaging Technique in the Mechanics of Suspensions", ASME FED - Vol. 243, pp. 41-60.

In this paper authors compared and analyzed the two most advanced theories of obtaining governing equations for heterogeneous media, namely - 1) volume averaging approaches, and ensemble averaging approaches. Referring to the volume averaging approaches, authors mention works made basically following to Ishii (1975), Delhaye (1981), Drew (1983) etc. It is strong opinion of the present authors that those works present incorrectly, inconsistent implementation and development of governing equations using volume averaging method. In lue of this the further comparisons of volume averaged governing equations mostly presented in the works by mentioned workers and their co-authors and equations obtained with the ensemble averaging techniques is in no interest here.

Their methodology of volume averaging will be commented on in analysis of Lahey and Drew studies below.

The second and primary method of heterogeneous media governing equations development addressed in the work by Buyevich and Theofanous (1997) is the ensemble averaging approach. Authors referring to works by themselves (Belousov, et al. 1985; Buyevich, 1992, 1995; Buyevich and Markov, 1973; Buyevich and Shchelchkova, 1978; Buyevich and Ustinov, 1992), and others as Joseph and Lundgren (1990), Zhang and Prosperetti (1994, 1997), etc.

While concerning with the obvious mismatching of the governing equations obtained through the two different approaches authors write that "the confusion due to various, seemingly incongruent, forms of the field equations (particularly of the momentum equations)" ... "this is especially troublesome, and we agree. It is not exactly clear what the practical impacts are, but such a confusion goes to the heart of one's educational effort and undermines the very foundation of the field".

In the development of the governing equations for the suspensions of the fine particles authors used to rely on the techniques advanced in the previous works and the following sag forces exerted by the ambient fluid are linear in the relative fluid velocity (which is applicable to only finest particle suspensions);

2) "all surface tension effects are ignored, so that stresses have no discontinuity at the interface";

3) "overlook possible contributions to the effective stresses acting in mean suspension flow of fluctuations of both spheres and fluid, such as the Reynolds stresses";

4) "spheres to be free from imbedded dipole moments, so that there is no dipole interaction of the spheres between themselves and with a corresponding external field";

5) the relevant and interactive fields of particles positions, velocities, accelerations and angular velocities are taken as independent variables, so that the "strong friction" approximation undertaken to simplify the development, which implies that

the only "the positions vectors alone, ... are quite sufficient to characterize possible configurations of the particulate ensemble";

6) terms caused by fluctuations are neglected;

7) taken monodisperse suspension as for the purpose not to treat seems unrealistically difficult polydisperse ensembles.

As some of the assumptions listed above are rather justified for the finest particles - 1, 2, 4, while the assumptions 3, 5, 6, and 7 are adopted seems for only purpose to overcome the difficulties in mathematical procedures. Also assumptions 1, 3, 5, and 6 mean that the linear problems are only considered, even when convective phenomena have being treated.

The closure of derived equations is obtained in a fashion that brings the ideas when "the model of homogeneous effective medium is approximately valid". That means that the problem's solution is thought through the "conditional averages over the dispersed phase must be found to calculate the necessary integrals, which implies solving problems that concern flow and heat transfer near the test sphere". This last statement implies that the closure should be obtained through the numerical or experimental solution of the volume test problem. Which is nothing more than the effective medium approximation when medium phenomena around the selected particle are simulated by pretending that the characteristics of medium are as they must be if the problem is solved. Another remark concerning the closure in the ensemble averaging approach is that in the case of inhomogeneous media or, for example, polydisperse discontinuous phase distribution one needs to model and simulate the multibody polysize closure problem to the full of its extent.

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It worth and relevent to state here that the VAT at present time is developed to the level that all the above mentioned assumed restrictions can be treated in VAT through the proper mathematical procedures.

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0.0.1 Analysis of separate terms in averaged equations.

In the Buyevich and Teofanous (1997):

Also they stated in (35) that following is true

$$\langle \theta_2 \nabla \cdot (DC_i \mathbf{C}) \rangle = \nabla \cdot \langle \theta_2 DC_i \mathbf{C} \rangle \ , \ (35) \tag{1}$$

where θ_2 is the

D is the

C_i is the

$\mathbf{C}$ is the

And at the same time for the averaged gradient of the momentum stress tensor (47) they arrived to

$$\left\langle \theta_2 \nabla \cdot \widehat{\mathbf{\Sigma}} \right\rangle = n \oint \mathbf{n} \cdot \left\langle \widehat{\mathbf{\Sigma}} \right\rangle_{\mathbf{r}} d\mathbf{a}, (\mathbf{47}) \tag{2}$$

but not having the term

$$\left\langle \theta_2 \nabla \cdot \widehat{\mathbf{\Sigma}} \right\rangle \neq \nabla \cdot \left\langle \theta_2 \widehat{\mathbf{\Sigma}} \right\rangle .? \tag{3}$$

Also

$$\left\langle \theta_2 \widehat{\mathbf{E}} \right\rangle = n \oint \frac{1}{2} \left(\mathbf{n} * \langle \mathbf{C} \rangle_{\mathbf{r}} + \langle \mathbf{C} \rangle_{\mathbf{r}} * \mathbf{n} \right) d\mathbf{a}, (40) \tag{4}$$

where $\widehat{\mathbf{E}}$ is the
while

$$\left\langle \theta_2 \widehat{\mathbf{\Sigma}} \right\rangle = n \left\{ \oint \mathbf{a} * \left(\mathbf{n} * \left\langle \widehat{\mathbf{\Sigma}} \right\rangle_{\mathbf{r}} \right) d\mathbf{a} - \int\limits_{x \leq a} \mathbf{x} * \left(\nabla \cdot \left\langle \widehat{\mathbf{\Sigma}} \right\rangle_{\mathbf{r}} \right) d\mathbf{x} \right\} .(46) \tag{5}$$

Here in (1, 4, 5, 2) are the major discrepencies between the VAT and ensemble averaging method in Buyevich's implementation.

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In averaging of the strain tensor in the continuous phase the result is

$$\left\langle \theta_1 \widehat{\mathbf{E}} \right\rangle = \left\langle \widehat{\mathbf{E}} \right\rangle = \widehat{\mathbf{e}} = \frac{1}{2} \left\| \frac{\partial V_i}{\partial r_k} + \frac{\partial V_k}{\partial r_i} \right\|, \tag{6}$$

where $\widehat{\mathbf{e}}$ is the "strain tensor for mean suspension flow"?

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Also, the remark concerning the diffusion term ensemble averaging as it's done by Buyevich and Teofanous (1997). In the second equation in (52) (for dispersed phase) using the second equation in (25) (by Buyevich and Teofanous, 1997 also) and (1)

$$\langle \theta_2 \nabla \cdot \mathbf{Q} \rangle = \nabla \cdot \langle \theta_2 \mathbf{Q} \rangle = \{ \lambda_2 \langle \theta_2 \Delta T \rangle \} \approx \lambda_2 n \oint \nabla \langle T \rangle \cdot \vec{n} da, \tag{7}$$

where curly brackets meaning the averaging (volume) over the dispersed medium and direction of the normal vector n under the integral sign is of no matter (see comments to expressions (46) and (47) by Buyevich and Teofanous, 1997).

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Making direct comparison of mathching terms in works by Buyevich and coauthors as, for example, in Buyevich and Korneev (1974) article gives more precise look on the compared methods.
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The averaged diffusivity term in the right hand side of the equation of thermal transport in the entire medium without source terms is (Buyevich and Korneev, 1974)

$$= -\nabla \mathbf{q} = -\nabla \left[\langle \mathbf{q}_f \rangle_f + \langle \mathbf{q}_s \rangle_s \right] = -\nabla \left[\lambda_0 \langle -\nabla T_f \rangle_f - \lambda_1 \langle \nabla T_s \rangle_s \right] = \tag{8}$$

$$= -\nabla \left[\lambda_0 \langle -\nabla T_f \rangle_f - \lambda_0 \langle \nabla T_s \rangle_s + \lambda_0 \langle \nabla T_s \rangle_s - \lambda_1 \langle \nabla T_s \rangle_s \right] = \tag{9}$$

$$= -\nabla \left[\lambda_0 \langle -\nabla T \rangle - (\lambda_1 - \lambda_0)\, n \int\limits_{\partial S_{wp}} \{T_s\}_s \; \vec{ds}_1 \right], \tag{10}$$

where n is the number concentration function (here n is the number of particles in a selected subvolume (actually meaning in the REV)), ∂S_{wp} is the surface of one particle, so the integration is provided over the surface of one test particle (actually meaning the assumption of the effective medium approximation). This expression can be also written as

$$= -\nabla \left[\lambda_0 \langle -\nabla T \rangle - (\lambda_1 - \lambda_0)\, n \int\limits_{\partial S_{wp}} \{T_f\}_f \; \vec{ds}_1 \right], \tag{11}$$

as long as averaged temperature on the particle's surface assumed to be equal. Averaged values comply to the equality

$$\langle \nabla T \rangle = \langle \nabla T_f \rangle_f + \langle \nabla T_s \rangle_s \,. \tag{12}$$

Also the notations in the above expressions are following the convenient system established in the volume averaging theory. It is worth to admit that the notations of the VAT quite applicable here as long as in the work by Buyevich and Korneev (1974) used the volume averaging approach. Also, as remarked later in works by Buyevich and co-authors (see, for example, Buyevich et al., 1976; and Buyevich and Perminov, 1980) for the adopted conditions of the problem formulation in the ensemble averaging approach for modeling of two-phase medium transport the equations of heat transport are complitely coinside with the volume averaged equations (assuming that the same conditions applied to both models).

The fully written right hand side in the equation of continuous phase (taken without source term, which is of no interest here) consists of

$$= -\nabla \left[\lambda_0 \langle -\nabla T \rangle - (\lambda_1 - \lambda_0)\; n \int\limits_{\partial S_{wp}} \{T_s\}_s \; \vec{ds}_1 \right] + n \int\limits_{\partial S_{wp}} \left(\lambda_0 \{\nabla T_f\}_f \right) \; \vec{ds}_1, \tag{13}$$

where the last term can be taken as the flux determined in the continuous phase. At the the same time this flux is not determined in the paper for the solid phase, because the right hand part of the particles temperature expressed as

$$= -n \int\limits_{\partial S_{wp}} \left(\lambda_0 \ \left\{ \nabla T_f \right\}_f \right) \overrightarrow{ds}_1, \tag{14}$$

which is the last term in the previous equation taken with opposite sign.

In later paper by Buyevich et al. (1976) for the same medium of dispersed particles were derived the equations of heat and mass transport by means of ensemble averaging technique. Those two sets of equations in paper by Buyevich and Perminov (1980) were acknoweledged as being equal and written in closed form for the continuous phase

$$\langle m \rangle \left(c\rho \right)_f \left(\frac{\partial}{\partial t} + V \nabla \right) \left\{ T_f \right\}_f = \lambda_{eff} \Delta \left\{ \nabla T_f \right\}_f - \beta \left(\left\{ T_f \right\}_f - \left\{ T_s \right\}_s \right), \tag{15}$$

and for dispersed phase as

$$\langle s \rangle \left(c\rho \right)_s \frac{\partial \left\{ T_s \right\}_s}{\partial t} = \beta \left(\left\{ T_f \right\}_f - \left\{ T_s \right\}_s \right), \tag{16}$$

where β is the coefficient of interface heat transfer.

Comparing the equations (13, 14) and right hand sides in the equations (15, 16) one can observe the extent of adopted physically correct but not mathematically justified closure method in which proclaimed adequacy of these expressions.

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The above analysis concerning the volume averaging method applied by Buyevich and Korneev (1974) can be summarized as that:

the method of volume averaging by these authors **was developed incorrectly** (even notwithstanding the neglecting of deviation terms) and consequently gave incorrect final averaged equations, those were later reappeared in the ensemble averaged development. For example, the equation for dispersed phase have no physical mechanism to express the intra phase heat conduction, no matter of the particle size or time scale considered.

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The most arguable (determined, compelling) comparison should be made with the corresponding (meaning developed with the same assumptions) VAT governing equations

$$\langle m \rangle \left(\varrho c_p \right)_f \left(\frac{\partial \widetilde{T}_f}{\partial t} + \widetilde{U}_i \nabla \widetilde{T}_f \right) = k_f \nabla \nabla \left(\langle m \rangle \widetilde{T}_f \right) +$$
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$$+k_f \nabla \cdot \left[\frac{1}{\Delta\Omega} \int\limits_{\partial S_w} T_f \vec{ds} \right] + \frac{k_f}{\Delta\Omega} \int\limits_{\partial S_w} \nabla T_f \cdot \vec{ds}, \tag{17}$$

$$\begin{aligned} \langle s \rangle \left(\varrho c_p\right)_s \frac{\partial \widetilde{T}_s}{\partial t} &= \nabla^2 \left(\langle s \rangle \widetilde{T}_s \right) + k_s \nabla \cdot \left[\frac{1}{\Delta\Omega} \int\limits_{\partial S_w} T_s \vec{ds_1} \right] + \\ &+ \frac{k_s}{\Delta\Omega} \int\limits_{\partial S_w} \nabla T_s \cdot \vec{ds_1}. \end{aligned} \tag{18}$$

where

$$\widetilde{T}_f = \{T_f\}_f \, . \tag{19}$$

Those equations after closure of the last heat exchange term (Travkin and Catton, 1992, 1995) will be

$$\begin{aligned} &\langle m \rangle \left(\varrho c_p\right)_f \left(\frac{\partial \widetilde{T}_f}{\partial t} + \widetilde{U}_i \nabla \widetilde{T}_f \right) = k_f \nabla\nabla \left(\langle m \rangle \widetilde{T}_f \right) + \\ &+ k_f \nabla \cdot \left[\frac{1}{\Delta\Omega} \int\limits_{\partial S_w} T_f \vec{ds} \right] + \widetilde{\alpha}_T S_w \left(\{T\}_s - \{T\}_f \right) = \\ &= T_f D_1 + T_f M D_1 + T_f M E_1. \end{aligned} \tag{20}$$

$$\begin{aligned} &\langle s \rangle \left(\varrho c_p\right)_s \frac{\partial \ \{T\}_s}{\partial t} = \nabla^2 \left(\langle s \rangle \widetilde{T}_s \right) + k_s \nabla \cdot \left[\frac{1}{\Delta\Omega} \int\limits_{\partial S_w} T_s \vec{ds_1} \right] + \\ &+ \widetilde{\alpha}_T \ S_w \left(\{T\}_f - \{T\}_s \right) = \\ &= T_s D_1 + T_s M D_1 + T_s M E_1. \end{aligned} \tag{21}$$

Comparison of these two equations with the Buyevich and Perminov's (1980) (15, 16) equations gives the difference in each term. Meanwhile, there is some good general resemblance of the terms evidently shown between (13, 14) and terms in the right hand side of VAT's (17), (18)

$$\left[-\nabla \left[\lambda_0 \left\langle -\nabla T \right\rangle - (\lambda_1 - \lambda_0)\ n \int\limits_{\partial S_{wp}} \{T_s\}_s\ \vec{ds_1} \right] + n \int\limits_{\partial S_{wp}} \left(\lambda_0 \left\{ \nabla T_f \right\}_f \right)\ \vec{ds_1} \right] \sim$$

$$k_f \nabla\nabla \left(\langle m \rangle \widetilde{T}_f \right) + k_f \nabla \cdot \left[\frac{1}{\Delta\Omega} \int\limits_{\partial S_w} T_f \vec{ds} \right] + \frac{k_f}{\Delta\Omega} \int\limits_{\partial S_w} \nabla T_f \cdot \vec{ds} \tag{22}$$

and

$$\left[-n \int\limits_{\partial S_{wp}} \left(\lambda_0\ \left\{ \nabla T_f \right\}_f \right) \vec{ds_1} \right] \sim$$

$$\frac{k_s}{\Delta\Omega} \int\limits_{\partial S_w} \nabla T_s \cdot \vec{ds_1}, \tag{23}$$

where

$$\lambda_0\ \neq k_s,\ \lambda_0\ = k_f.$$

The importance of keeping all the required terms in the averaged governing equations was recently demostrated by Travkin and Kushch (1998) while comparing the strict solution of the VAT heat conductance equations (as just presented above, only in steady state case) and with the direct numerical solution of the multibody spatially heterogeneous problem for the same globular morphology.

The coincidence of the results of the exact calculation of two equation three term energy transport VAT model with the exact DNM of the one-temperature effective coefficient model for heterogeneous media with non-constant spatial morphology established confidence in necessity of using all the terms in the VAT equations.

The need for the morpho-diffusive terms in the energy equations (20, 21) are further demonstrated by noting that their magnitudes - for example, diffusion term in disperse medium equation $T_s D_1$ and morpho-diffusive terms $T_s M D_1$ and $T_s M E_1$ are all of the same value rate of above 10 % of the total absolute budget.

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0.0.2 Concluding comments on Buyevich's papers and his conclusions in (1997) paper

Concerning the introduction of "additional ergodic hypotheses about the equivalence of averaging over volume and averaging over a surface", as was stated in Buyevich et al. (1976) it needs to be said that this is only the working closure hypothesis used in some closure models and it's not worse than other hypothesis which were adopted
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for the sake of closure idea while developing resonable solution algorithms - see, for example, works by Buyevich himself (1974, 1980, 1989) and many works in two-phase transport modeling.

In the conclusion authors are stressed that:

2) "The ensemble averaging does not require the length scale for mean variables to be large as compared with a linear size of a small physical volume that nevertheless must contain a great number of particles, needed for volume averaging".

Meanwhile, in the same article the scale assumptions are provided, see (14), and generally ensemble averaging can be accomplished not only through an ensemble but and via the physical scale averaging, for example, in closure procedures. Also, if the effective medium approximation is no longer assumed as the valid approximation method, then the whole ensemble realization needs to be considered.

3) "In contrast to time and volume averaging, no difficulty arises in ensemble averaging with respect to discontinuity of derivatives of averaged quantities".

If any feature like discontinuity of the derivatives appears in the model due to physical necessity as, for example, temperature derivatives near the interface in each of the side are different, so the averaged variables should be of the same character - thus having discontinuity. All other artificial difficulties arise due to improper volume averaging theory model development and such shortcuts as interface integrals or fluctuation terms being simulated as with some continuous approach or by using elimination of important features.

In some models developed (Lahey and Drew, 1988) the functions or terms are considered existing on the interface surface and at the same time being not necessarily equal on the both sides of the interface. While in some (many in two-phase transport) that is the truth for some fields as a density or velocity, for example, while for another fields as assigned fields functions (often in tensorial form) they are just heuristic assumptions not supported by strict VAT theorems. Additional physics, as a rule needs to be considered.

Also, and this is very important, the necessity and compulsion of the jump boundary conditions lies in the heart of the heterogeneous medium transport phenomena (physics) - because the local fields quantities and the non-local fields in the vicinity of interface surface are not necesseraly the equal physical quantities (see, for example, early works by Khoroshun, 19 ; also study by Whitaker and Ochoa-Tapia, 1998; also Travkin and Catton, 1999.

4) "An essential uncertainty is known to arise in volume averaging as to how quantities obtained by averaging over a representative volume and by averaging over differently oriented surfaces correlate between themselves. Any hypothesis that states the surface averages being dependent on surface orientation leads to a conclusion that effective stresses must be not symmetrical. .... The ensemble averaging technique is entirely free of this deficiency".

The averaged over the REV and interface surface variables not need to be in some specific correlation interdependency, they are simply different variables, those which have interconnection, but of the same nature as any other varaible or function appearing in the model. Introduced firstly by Banerjee (1980) this kind of averaged quantities became accustomary in works by Lahey and Drew (see, for example, Lahey and Drew, 1988; etc.). Also, if the physical field limiting values at the both sides of the interface surface are different due to physics of the problem - as in case of concentration near the vapour-liquid interface, and different are their derivatives, as, for example are stress tensors or velocities, then there is not to be a confusion regarding the treatment of these entities in each of the media (fluids).

The problem is that each time someone is about to introduce the new variable - it means that this variable should be treated as a physical variable and got closure modeling along with the initially set up variables.

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0.0.3 Buyevich, Y.A., Ustinov, V.A., and Khuzhaerov, B., (1989), "Nonsteady Transfer in Disperse and Heterogeneous Media", Inz.-Fiz. Zh., Vol. 56, No. 5, pp. 779-787 (in Russian).

Given a good closure method for the case of uniform dispersed medium two-temperature transient heat transfer. The equations are adopted in reduced non-fluctuation form.But the boundary conditions taken traditional - III-rd, I-st and II-nd kinds. This closure idea can be used in other problems when mathematical statement is reduced to the non-fluctuation problem.

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Buyevich, Y.A., Korneev, Y.A., and Shchelchkova, I.N., (1976), "Transport of Heat or Mass in Disperse Flow", *Inz.-Fiz. Zh.*, Vol. 30, No. 6, pp. 979-985 (in Russian).

0.0.4 Buyevich, Yu. A., Korneev, Yu. A., Schelchkova, I. N. (1976), "Transport of Heat

or Mass in Disperse Flow," Journal of Engineering Physics, Vol. 30, No. 6, pp. 635-640.

In this work for the first time was used the method of ensemble averaging technique to obtain the two-temperature mathematical statement for the heat transport in dispersed media. Convective term involved fluctuations of the velocity and temperature fields were taken into account.

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Buyevich, Y.A. and Perminov, E.B., (1980), "Nonstationary Heating of a Fixed Granular Mass", *Inz.-Fiz. Zh.*, Vol. 38, No. 1, pp. 29-37 (in Russian).

0.0.5 Buyevich, Yu. A. and Perminov, E. B. (1980), "Nonstationary Heating

of a Fixed Granular Mass," Journal of Engineering Physics, Vol. 38, No. 1, pp. 19-25.

In this work given the solution of two-temperature averaged equations with heat exchange term and in non-fluctuation llinear statement for a granular medium
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using perturbation method. Resulting one temperature equation contains an infinite chain of time derivatives.

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0.0.6 Buyevich, Yu. A., Korneev, Yu. A. (1974), "On Heat and Mass Transfer in Disperse Medium," Prikl. Mech. Tech. Phys., No. 4, pp. 79-87 (in Russian).

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0.0.7 Belousov, V.S., Buyevich, Y.A., and Yasnikov, G.P., (1985), "The Method of Trajectory Integrals in the Fluid Dynamics of Suspensions", Inzh.-Fiz. Zh., Vol. 48, pp. 602-609 (in Russian).

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0.0.8 Buyevich, Y.A., (1992), "Heat and Mass Transfer in Disperse Media. Parts I and II", Int. J. Heat Mass Transfer, Vol. 35, pp. 2445-2463.

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0.0.9 Buyevich, Y.A., (1995), "Interphase Interaction in Fine Suspension Flow", Chem. Engng. Sci., Vol. 50, pp. 641-650.

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0.0.10 Buyevich, Y.A. and Markov, V.G., (1973), "Continuum Mechanics of Monodisperse Suspensions. Parts 1 and 2", Prikl. Matem. Mekh., Vol. 37, pp. 882-894, 1059-1077 (in Russian).

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0.0.11 Buyevich, Y.A. and Shchelchkova, I.N., (1978), "Flow of Dense Suspensions", Progr. Aerospace Sci., Vol. 18, pp. 121-150.

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0.0.12 Buyevich, Y.A. and Ustinov, V.A., (1992), "Non-stationary Heat Transfer and Dispersion Effects in Granular Media", Int. J. Heat Mass Transfer, Vol. 35, pp. 2435-2444.

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0.0.13 Buyevich, Yu. A., Korneev, Yu. A. (1976), "Dispersion of Thermal Waves in Granular Material," Journal of Engineering Physics, Vol. 31, No. 1, pp. 766-772.

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