0.0.1 POROUS MEDIUM TURBULENT VAT EQUATIONS

After averaging over the REV the basic initial set of turbulent transport equations (see, for example, Rodi [27])

$$\frac{\partial \overline{U}_i}{\partial t} + \overline{U}_j \frac{\partial \overline{U}_i}{\partial x_j} = -\frac{1}{\varrho_f} \frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \overline{U}_i}{\partial x_j} - \overline{u_i' u_j'} \right) + S_{U_i}, \tag{1}$$

$$\frac{\partial \overline{\Phi}_f}{\partial t} + \overline{U}_j \frac{\partial \overline{\Phi}_f}{\partial x_j} = \frac{\partial}{\partial x_j} \left(D_f \frac{\partial \overline{\Phi}_f}{\partial x_j} - \overline{u_i' \varphi_f'} \right) + S_{\Phi_f}, \tag{2}$$

$$\frac{\partial \overline{U}_i}{\partial x_i} = 0, \tag{3}$$

and using the averaging formalism developed in the works by Primak et al. [22], Shcherban et al.[23], Primak and Travkin [28], one obtains the following equations for mass conservation

$$\frac{\partial}{\partial x_i} \left\langle \overline{U}_i \right\rangle_f + \frac{1}{\Delta\Omega} \int\limits_{\partial S_w} \overline{U}_i \cdot \vec{ds} = 0, \tag{4}$$

for turbulent filtration (with molecular viscosity terms neglected for the simplicity)

$$\begin{aligned}
&\langle m \rangle \frac{\partial \widetilde{\overline{U}}_i}{\partial t} + \frac{\partial}{\partial x_j} \left(\langle m \rangle \widetilde{\overline{U}}_j \widetilde{\overline{U}}_i \right) = \\
&-\frac{1}{\varrho_f} \frac{\partial}{\partial x_i} \left(\langle m \rangle \ \widetilde{\overline{p}} \right) + \frac{\partial}{\partial x_j} \left\langle -\overline{u_j' u_i'} \right\rangle_f + \frac{\partial}{\partial x_j} \left\langle -\widehat{\overline{u}}_j \widehat{\overline{u}}_i \right\rangle_f - \\
&-\frac{1}{\varrho_f \Delta\Omega} \int\limits_{\partial S_w} \overline{p} \cdot \vec{ds} - \frac{1}{\Delta\Omega} \int\limits_{\partial S_w} \overline{U}_j \overline{U}_i \cdot \vec{ds} - \\
&-\frac{1}{\Delta\Omega} \int\limits_{\partial S_w} - \overline{u_j' u_i'} \cdot \vec{ds} + \langle m \rangle \widetilde{S}_{U_i}, \ i,j = 1-3,
\end{aligned} \tag{5}$$

and for scalar diffusion (with molecular diffusivity terms neglected)

$$\begin{aligned}
&\langle m \rangle \frac{\partial \widetilde{\overline{\Phi}}_f}{\partial t} + \frac{\partial}{\partial x_i} \left(\langle m \rangle \widetilde{\overline{U}}_i \widetilde{\overline{\Phi}}_f \right) = \\
&\frac{\partial}{\partial x_i} \left\langle -\overline{u_i' \varphi_f'} \right\rangle_f + \frac{\partial}{\partial x_i} \left\langle -\widehat{\overline{u}}_i \widehat{\overline{\varphi}}_f \right\rangle_f - \\
&-\frac{1}{\Delta\Omega} \int\limits_{\partial S_w} \overline{U}_i \overline{\Phi}_f \cdot \vec{ds} -
\end{aligned}$$

$$-\frac{1}{\Delta\Omega}\int\limits_{\partial S_w}\overline{u_i'\varphi_f'}\cdot\overrightarrow{ds}+\langle m\rangle\,\widetilde{S}_{\Phi_f},\ i=1-3. \qquad (6)$$

Many details and possible variants of the above written equations with tensorial terms are found in Primak et al. [22], Shcherban et al. [23], Travkin and Catton [16,17]. The volume averaging procedures were applied in the work by Masuoka and Takatsu [29] to derive the VAT turbulent transport equations. Like in numerous other studies pertaining to multiphase transport modeling, the major difficulty in averaging right hand side terms has been overcome using assumed artificial closure models for stress components terms. As a result the averaged turbulent momentum equation, for example, has conventional additional resistance terms like the momentum averaged equation developed by Vafai and Tien [30] for laminar regime transport in porous medium.

One dimensional mathematical statements will be used in what follows further for simplicity. Admission of specific types of medium irregularity or randomness requires that complicated additional expressions be included in the generalized governing equations. Treatment of these additional terms becomes a crucial step once the governing averaged equations are written. An attempt to implement some basic departures from a porous medium with strictly regular morphology descriptions into a method for evaluation of some of the less tractable, additional terms is explained below.

The 1-D momentum equation with terms representing a detailed description of the medium morphology is depicted as follows

$$\frac{\partial}{\partial x}\left(\langle m\rangle\left(\widetilde{K}_m+\nu\right)\frac{\partial\widetilde{\overline{U}}}{\partial x}\right)+\frac{\partial}{\partial x}\left(\left\langle\widehat{K}_m\frac{\partial\,\widehat{\overline{u}}}{\partial x}\right\rangle_f\right)+\frac{\partial}{\partial x}\left(\left\langle-\widehat{\overline{u}}\,\widehat{\overline{u}}\right\rangle_f\right)=$$

$$\langle m\rangle\,\widetilde{\overline{U}}\frac{\partial\,\widetilde{\overline{U}}}{\partial x_j}-\frac{1}{\Delta\Omega}\int\limits_{\partial S_w}\left(K_m+\nu\right)\frac{\partial\overline{U}}{\partial x_i}\cdot\overrightarrow{ds}+\frac{1}{\varrho_f\Delta\Omega}\int\limits_{\partial S_w}\overline{p}\overrightarrow{ds}+\frac{1}{\varrho_f}\frac{\partial}{\partial x}\left(\langle m\rangle\,\widetilde{\overline{p}}\right)=$$

$$\langle m\rangle\,\widetilde{\overline{U}}\frac{\partial\,\widetilde{\overline{U}}}{\partial x_j}+u_{*rk}^2S_w\left(x\right)+\frac{1}{\varrho_f\Delta\Omega}\int\limits_{\partial S_w}\overline{p}\overrightarrow{ds}+\frac{1}{\varrho_f}\frac{\partial}{\partial x}\left(\langle m\rangle\,\widetilde{\overline{p}}\right), \qquad (7)$$

where K_m is the turbulent eddy viscosity, u_{*rk}^2 is the square friction velocity at the upper boundary of surface roughness layer h_r averaged over interface surface S_w.

General statements for energy transport in a porous medium require two-temperature treatments. Travkin et al. [31] showed that the proper form for the turbulent heat transfer equation in the fluid phase using K-theory one-equation closure with primarily 1-D convective heat transfer is

$$c_{pf}\varrho_f \langle m\rangle \widetilde{\overline{U}}\frac{\partial\widetilde{\overline{T}}_f}{\partial x} = \frac{\partial}{\partial x}\left[\langle m\rangle \left(\widetilde{K}_T + k_f\right)\frac{\partial\widetilde{\overline{T}}_f}{\partial x}\right] +$$

$$+\frac{\partial}{\partial x}\left(\left\langle \widehat{K}_T \frac{\partial\,\widehat{\overline{T}}_f}{\partial x}\right\rangle_f\right) + c_{pf}\varrho_f \frac{\partial}{\partial x}\left[\langle m\rangle \left\{-\widehat{\overline{T}}_f\ \widehat{\overline{u}}\right\}_f\right] +$$

$$+\frac{\partial}{\partial x}\left[\frac{\left(\widetilde{K}_T + k_f\right)}{\Delta\Omega}\int\limits_{\partial S_w}\widehat{\overline{T}}_f \vec{ds}\right] + \frac{1}{\Delta\Omega}\int\limits_{\partial S_w}\left(K_T + k_f\right)\frac{\partial\overline{T}_f}{\partial x_i}\cdot\vec{ds}, \tag{8}$$

while in the neighboring solid phase, the corresponding equation is

$$\frac{\partial}{\partial x}\left[(1-\langle m\rangle)\left\{K_{sT}\right\}_s \frac{\partial\left\{T_s\right\}_s}{\partial x}\right] + \frac{\partial}{\partial x}\left(\left\langle \widehat{K}_{sT}\frac{\partial\,\widehat{T}_s}{\partial x}\right\rangle_s\right) +$$

$$+\frac{\partial}{\partial x}\left[\frac{\left\{K_{sT}\right\}_s}{\Delta\Omega}\int\limits_{\partial S_w}\widehat{T}_s \vec{ds}_1\right] + \frac{1}{\Delta\Omega}\int\limits_{\partial S_w} K_{sT}\frac{\partial T_s}{\partial x_i}\cdot\vec{ds}_1. \tag{9}$$

The generalized longitudinal 1-D mass transport equation in the fluid phase, including description of potential morpho-fluctuation influences, for a medium morphology with only 1-D fluctuations is written

$$\frac{\partial}{\partial x}\left[\langle m\rangle\left(\widetilde{K}_C + D_f\right)\frac{\partial\widetilde{\overline{C}}_f}{\partial x}\right] + \frac{\partial}{\partial x}\left(\left\langle \widehat{K}_C\frac{\partial\,\widehat{\overline{C}}_f}{\partial x}\right\rangle_f\right) +$$

$$+\frac{\partial}{\partial x}\left[\langle m\rangle\left\{-\widehat{\overline{c}}_f\ \widehat{\overline{u}}\right\}_f\right] + \frac{\partial}{\partial x}\left[\frac{\left(\widetilde{K}_C + D_f\right)}{\Delta\Omega}\int\limits_{\partial S_w}\widehat{\overline{c}}_f \vec{ds}\right] +$$

$$+\frac{1}{\Delta\Omega}\int\limits_{\partial S_w}\left(K_C + D_f\right)\frac{\partial\overline{C}_f}{\partial x_i}\cdot\vec{ds}\ + \langle m\rangle\widetilde{S}_C = \langle m\rangle\widetilde{\overline{U}}\frac{\partial\widetilde{\overline{C}}_f}{\partial x}, \tag{10}$$

while the corresponding nonlinear equation for the solid phase

$$\frac{\partial}{\partial x}\left[(1-\langle m\rangle)\left\{D_s\right\}_s\frac{\partial\left\{C_s\right\}_s}{\partial x}\right] + \frac{\partial}{\partial x}\left(\left\langle \widehat{D}_s\frac{\partial\,\widehat{c}_s}{\partial x}\right\rangle_s\right) +$$

$$+\frac{\partial}{\partial x}\left[\frac{\left\{D_s\right\}_s}{\Delta\Omega}\int\limits_{\partial S_w}\widehat{C}_s\vec{ds}_1\right] + \frac{1}{\Delta\Omega}\int\limits_{\partial S_w} D_s\frac{\partial C_s}{\partial x_i}\cdot\vec{ds}_1. \tag{11}$$