0.0.1 Analysis of Pressure Loss Homogeneous Experimental Data for Porous Media Morphologies Based on VAT

Ergun [76] suggested two types of effective bulk friction factors. One of them, the so-called kinetic energy friction factor f_{ker} is similar to the Fanning friction factor, f_f , and is written with the same assumptions,

$$f_f = \frac{f_{ker}}{3} = \frac{d_h}{2\,\rho_f\,\widetilde{\overline{U}}^2}\left(\frac{\Delta P}{L}\right), \tag{1}$$

where d_h is the hydraulic diameter and $\widetilde{\overline{U}}$ the intrinsic averaged velocity including turbulent regime. The problem is what to choose for the hydraulic diameter for a given porous media that properly represents its morphology. Bird et al. [77] used the ratio of the volume available for flow to the cross section available for flow in their derivation of a hydraulic radius r_{hb}. This assumption led them to

$$r_{hb} = \frac{\langle m\rangle\, d_p}{6(1-\langle m\rangle)},$$

where d_p is the particle diameter. A basis for a consistant hydraulic diameter for any systems is

$$d_h = \frac{4\,\langle m\rangle}{S_w} = \frac{4\,\langle m\rangle}{a_v(1-\langle m\rangle)} = \frac{2\,\langle m\rangle}{3(1-\langle m\rangle)}d_p = 4r_{hb}, \tag{2}$$

where a_v is the particle specific surface which is equal to the total particle surface divided by the volume of the particle and specific surface $S_w = a_v(1-\langle m\rangle)$.This expression is justified when an equal or mean particle diameter is $d_p = 6/a_v$, which is exact for spherical particles and often used as a substitute for granular media particles. The value of hydraulic radius given by Bird et al. [77] was chosen by Chhabra [78] and used in his determination of a specific friction factor for capillary models.

The particle diameter d_p is often used as a characteristic length when a particulate medium is of primary interest, as is done by Chhabra [78]. The friction factor used in his book is written in the form

$$f_{cb} = \frac{d_p}{\langle m\rangle^2\ \rho_f\,\widetilde{\overline{U}}^2}\frac{\Delta p}{L}.$$

This friction factor, the friction factor f_b, given by the Bird et al. [77] for a packed bed, see equation (6.4-1), the Fanning friction factor, f_f ,and the Ergun‘s kinetic energy friction factor, f_{ker} ,and related by the following

$$f_{cb} = 2f_b = \left(\frac{1-\langle m\rangle}{\langle m\rangle^3}\right) f_{ker} \;=\; 3\left(\frac{1-\langle m\rangle}{\langle m\rangle^3}\right) f_f\ . \tag{3}$$

All these models use different length scales. It is not clear from evaluation of experimental data which is the most appropriate. To address this, the momentum equation for turbulent flow of an incompressible fluid in a porous media based on a simplifed VAT (SVAT) and K-theory will be used to develop a consistant set of morphological properties and a characteristic length. The one dimensional form of the momentum equation can be simplified for a regular morphology medium with constant porosity to the form of equation which is written

$$-\frac{d\widetilde{\overline{p}}}{dx} = \left(c_f + c_{dp}\frac{S_{wp}}{S_w}\right)\left(\frac{S_w}{\langle m\rangle}\right)\frac{\rho_f\,\widetilde{\overline{U}}^2}{2} = c_d\left(\frac{S_w}{\langle m\rangle}\right)\frac{\rho_f\,\widetilde{\overline{U}}^2}{2}, \tag{4}$$

where c_f is the friction factor and c_{dp} the form drag, S_{wp} is actually the cross flow form specific surface. The drag terms are combined into a single total drag coefficient

$$c_d = \left(c_f + c_{dp}\frac{S_{wp}}{S_w}\right), \tag{5}$$

to model the flow resistence terms in the general simplified momentum VAT equation. The drag resistance can be evaluated for a homogeneous porous media from measurements of pressure drop,

$$-\frac{d\,\widetilde{\overline{p}}}{dx} = f_f\,\left(\frac{S_w}{\langle m\rangle}\right)\frac{\rho_f\,\widetilde{\overline{U}}^2}{2}. \tag{6}$$

It was shown by Travkin and Catton [17] that a good approximation for such media is

$$c_d \cong f_f,$$

where a bulk value of pressure loss coefficient f_f can be obtained from experimental correlations. There are a few reasons why these quantities are not identical. One of them is the neglect of media inflow, outflow and other pressure losses which are usually incorporated into the correlations for f_f. Another reason is the loss of geometric characteristics represented partially by the ratio of S_{wp}/S_w.

Some details of a media composed of globular morphologies can also be described in terms of specific surface S_w, porosity $\langle m\rangle$, and particle diameter d_p. For a spherical particle medium when

$$S_w = \frac{6(1-\langle m\rangle)}{d_p},\;\; d_h = \frac{2}{3}\frac{\langle m\rangle}{(1-\langle m\rangle)}d_p,$$

which in turn yields the same relationship between the equavalent pore diameter and their parameters, as found for a one diameter capillary morphology, and leads to

$$S_w = \frac{6(1-\langle m\rangle)}{d_p} = \frac{6(1-\langle m\rangle)}{\left(\frac{3}{2}\frac{(1-\langle m\rangle)}{\langle m\rangle}d_h\right)} = \frac{4\,\langle m\rangle}{d_h}. \tag{7}$$

This leads to a relationship for d_h that takes the form

$$d_h = \frac{4\,\langle m\rangle}{S_w}. \tag{8}$$

This form is consistant with two major morphologies, both capillary and globular, and incorporates two properties of the media, both void fraction and specific surface area. It has a solid theoretical basis at least for two types of canonical porous media morphology (straight capillary parallel pore morphology - SPPM and one diameter sphere globular morphology) and was arrived at with different theoretical reasonings by others in previous years like Kays and London [79].

There are a number of analysis and experimental studies of different porous media morphology leading to the use of a porous media Reynolds number of the form

$$Re_{por} = \frac{4\,\widetilde{\widetilde{U}} < m >}{\nu S_w}. \tag{9}$$

Although various porous media pressure resistance models are described by others (see, Bird et al. [77], Fand et al. [80], and Chhabra [78]), the above description for d_h and Re_{por} and allows the transformation and comparison of correlation equations and experimental results obtained for diverse morphology media and the use of various scaling. Also, it allows experimentally determined characteristics of the media to be related to the closure relationship derived from the VAT analysis.

It was shown by Travkin and Catton [4,17] that a straight equal diameter tube morphology model yields the morphology functions $S_w / < m >$ and d_h is obtained from the above expression. A c_d for such a capillary morphology with its corresponding skin friction coefficient is given by Travkin and Catton [17],

$$c_d = \widetilde{c}_d \cong \frac{f_D}{4}, \tag{10}$$

which relates the Darcy pressure loss coefficient $f_D$to the SVAT model for some capillary morphologies. The generalized two term quadratic Reynolds-Forchheimer equation,

$$\begin{aligned} \frac{\Delta P}{L} &= \alpha\mu\, \widetilde{\widetilde{U}}\langle m\rangle + \beta\rho_f\, \widetilde{\widetilde{U}}^2 \langle m\rangle^2, \\ \alpha &= \frac{1}{k_D}[\frac{1}{m^2}],\ \beta = \beta[\frac{1}{m}]. \end{aligned} \tag{11}$$

needs to be compared to the SVAT momentum equation with constant morphological characteristics and flow field properties using the only resistance coefficient $c_d$described above

$$\frac{\Delta P}{L} = c_d \left(\frac{S_w}{\langle m\rangle}\right) \frac{\rho_f\, \widetilde{\widetilde{U}}^2}{2},\ [\frac{Pa}{m}]. \tag{12}$$

To make a transition between the both equations one can rewrite first equation in terms of the second one

$$\left[\frac{2\alpha\mu\ \langle m\rangle}{\rho_f\ \widetilde{\widetilde{U}}} + 2\beta < m >^2\right] \frac{\rho_f\ \widetilde{\widetilde{U}}^2}{2} =$$

$$= \left[\frac{2\alpha\mu\ \langle m\rangle}{\rho_f\ \widetilde{\widetilde{U}}} \left(\frac{\langle m\rangle}{S_w}\right) + 2\beta < m >^2 \left(\frac{\langle m\rangle}{S_w}\right)\right] \left(\frac{S_w}{\langle m\rangle}\right) \frac{\rho_f\ \widetilde{\widetilde{U}}^2}{2}.$$

Observing the right hand side of this expression it is obvious that this is the Fanning friction factor which is thought for the equation of 1D SVAT momentum flow, or

$$c_d = f_f = \left[\frac{\alpha\mu}{\rho_f\ \widetilde{\widetilde{U}}} + \beta < m >\right] \left(\frac{2\langle m\rangle^2}{S_w}\right). \tag{13}$$

This equation should be used to correlate experimental data on friction factor for highly porous media written in terms of a generalized two term quadratic Reynolds-Forchheimer equation. In this case the equation for Fanning friction factor f_f following from above, can be written in the form

$$c_d = f_f = \frac{A}{Re_{por}} + B, \tag{14}$$

where

$$Re_{por} = \frac{4\ \widetilde{\widetilde{U}} < m >}{\nu S_w},$$

$$A = \frac{8\alpha < m >^3}{S_w^2},\ B = 2\beta\frac{\langle m\rangle^3}{S_w}. \tag{15}$$

If Ergun‘s correlation is taken in the following generally used notation

$$\frac{\Delta p}{L} = \left(150\frac{(1-\langle m\rangle)^2}{d_p^2 \langle m\rangle^3}\right) \mu \langle m\rangle \widetilde{\widetilde{U}} + \left(1.75\frac{(1-\langle m\rangle)}{d_p \langle m\rangle^3}\right) \varrho_f \langle m\rangle^2 \widetilde{\widetilde{U}}^2, \tag{16}$$

with some additional algebra it can be transformed to the

$$\frac{\Delta p}{L} = \left[300\frac{(1-\langle m\rangle)^2}{d_p^2 \langle m\rangle^2} \frac{\nu}{\widetilde{\widetilde{U}}} + 3.5\frac{(1-\langle m\rangle)}{d_p \langle m\rangle}\right] \left(\frac{\varrho_f \widetilde{\widetilde{U}}^2}{2}\right) =$$

$$= \left[\left(\frac{50(1-\langle m\rangle)}{\langle m\rangle}\right) \frac{1}{Re_p} + \frac{3.5}{6}\right] \left(\frac{S_w}{\langle m\rangle}\right) \frac{\varrho_f \widetilde{\widetilde{U}}^2}{2},$$

where the specific surface S_w and particle Reynolds number Re_p are used to be as

$$S_w = \frac{6(1-\langle m\rangle)}{d_p},\ Re_p = \frac{\widetilde{\widetilde{U}} d_p}{\nu}.$$

From this formula the following will result as an expression for Ergun drag resistance coefficient in terms of simplified VAT

$$f_{er} = \frac{A_p^*}{Re_p} + B_p^*,\ A_p^* = \left(\frac{50(1-\langle m\rangle)}{\langle m\rangle}\right),\ \ B_p^* = \frac{3.5}{6} = 0.583. \tag{17}$$

The same formulation can be written in terms of the hydraulic diameter Reynolds number $Re_{ch}$by noting that

$$\begin{aligned} Re_p &= \frac{\widetilde{\widetilde{U}} d_p}{\nu} = \frac{\widetilde{\widetilde{U}}}{\nu}\left[\frac{3\,(1-\langle m\rangle)}{2\,\langle m\rangle}\right] d_h = Re_{ch}\left[\frac{3\,(1-\langle m\rangle)}{2\,\langle m\rangle}\right], \\ Re_{ch} &= Re_{por} = \frac{\widetilde{\widetilde{U}} d_h}{\nu} = \frac{2}{3}\frac{\langle m\rangle}{(1-\langle m\rangle)}\frac{\widetilde{\widetilde{U}} d_p}{\nu} = \frac{2}{3}\frac{\langle m\rangle}{(1-\langle m\rangle)} Re_p, \end{aligned} \tag{18}$$

which leads to~~

$$f_{er} = \frac{A_{ch}^*}{Re_{por}} + B_{ch}^*,\ A_{ch}^* = \frac{100}{3} = 33.33,\ \ B_{ch}^* = B_p^* = 0.583. \tag{19}$$

These parameters are very close to those of the Klenov and Matros [81] correlation with $A = 36.3$ and $B = 0.45$ for spherical particles and $A = 37.6$ and $B = 0.585$ for cylindrical particles. The correlation for the same $'Ergun's'$

SVAT drag resistance coefficient based on Re_{por} was derived above in (14) while known coefficients are $\alpha, 1\beta$, as well as permeability $\langle m\rangle$ and specific surface S_w. The former can be assessed using

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$$\alpha = 150\frac{(1-\langle m\rangle)^2}{d_p^2\,\langle m\rangle^3},\ \beta = 1.75\frac{(1-\langle m\rangle)}{d_p\,\langle m\rangle^3}. \tag{20}$$

A large amount of data exists that demonstrates the insufficiencies of the Ergun‘s drag resistance correlation (16). Because it was developed for specific morphology - in particular, globular “granular“ media, an application of the Ergun correlation to arbitrary selected or at least with arbitrary chosen relationships between porosity $\langle m\rangle$, specific surface S_w and pore (particle) diameter d_h can be quite unsatisfactory.

The formula for drag resistance of a spherical particle bed derived from Watanabe [82] results in a drag resistance coefficient that corresponds to the present notation

$$f_w = c_{sph}\frac{25\,(1-\langle m\rangle)}{12} = c_{d,w}\ , \tag{21}$$

where $c_{sph}$is the drag resistance coefficient of a single isolated bead of diameter d_p. The drag resistance coefficient derived from the correlations of Fand and Thinakaran [83], for a large range of Reynolds numbers

$$10^{-5} < Re_p = \frac{\tilde{\tilde{U}} d_p}{\nu} <\approx 0(10^3),$$

is

$$c_{d,ft} = f_{ft} = \frac{2A_{ft}}{(9Re_{por})} + \frac{B_{ft}}{3}, \tag{22}$$

where A_{ft} and B_{ft} are given by Fand and Thinakaran [83].

The widely used and cited Kozeny-Carman pressure loss equation can be written in few forms depending on the length scale taken and on the friction factor chosen. A form which incorporates morphological parameters is

$$\frac{\Delta p}{L} = k_{kc}\mu \left(\frac{S_w^2}{\langle m\rangle^2}\right)\tilde{\tilde{U}}, \tag{23}$$

where the coefficient k_{kc} (Kozeny-Carman coefficient) has been found for some media to be between 4.5 - 5. There are studies arguing such a narrow range (see Fand et al., [80]). Permeability in the Darcy sense following this equation can be written as

$$k_D = \frac{\langle m\rangle^3}{k_{kc}S_w^2}. \tag{24}$$

When the particle diameter serves as the length scale then the Kozeny-Carman dependency is

$$\frac{\Delta p}{L} = \mu\left(\frac{180}{d_p^2}\right)\tilde{\tilde{U}}\left(\frac{(1-\langle m\rangle)^2}{\langle m\rangle^2}\right), \tag{25}$$

which is distinquished from the Blake-Kozeny equation only by factor 180 instead of 150 in the last equation. With the hydraulic diameter length scale d_h this equation looks much simpler

$$\frac{\Delta p}{L} = \frac{\mu}{\left(\frac{d_h^2}{80}\right)}\tilde{\tilde{U}}, \tag{26}$$

with the Darcy permeability value estimated as

$$k_D = \left[\frac{d_h^2}{80}\langle m\rangle\right]. \tag{27}$$

The Fanning friction factor based on the Kozeny-Carman drag resistance model has the simple equation

$$f_{f,kc}\left(Re_{por}\right) = \frac{40}{Re_{por}},\ Re_{por} = \frac{4\,\widetilde{\overline{U}} < m >}{\nu S_w}. \tag{28}$$

Meanwhile, another well-known high flow regime drag resistance coefficient of Burke-Plummer (Bird et al., [77], Chhabra, [78]) has the very plain equation if written in the form of a Fanning friction factor using d_h as the scale length (valid naturally and for the SVAT pressure loss equation (4))

$$f_{f,bp} = \frac{1.75}{3} = 0.5833... \tag{29}$$

The Burke-Plummer friction loss model written in general d_p terms of the length scale d_p (equation (6.4-11) in Bird et al., [77], which was written rather intuitively as analogous to the more strict definition of capillary pressure loss model) is

$$\frac{\Delta p}{L} = f_{f,bp}\left(\frac{6}{d_p}\frac{(1-\langle m\rangle)}{\langle m\rangle}\right)\left(\frac{\varrho_f\,\widetilde{\overline{U}}^2}{2}\right). \tag{30}$$

Note, that this is exactly the SVAT drag resistance model (4) written in terms of globular (granular) media where it can usually be assumed that

$$\frac{S_w}{\langle m\rangle} = \frac{6}{d_p}\frac{(1-\langle m\rangle)}{\langle m\rangle}.$$

From equation (30) follows that the friction factor is

$$f_{f,bp} = \frac{d_p}{\varrho_f\,\widetilde{\overline{U}}^2}\left(\frac{\langle m\rangle}{3(1-\langle m\rangle)}\right)\frac{\Delta p}{L}, \tag{31}$$

which is just the Fanning friction factor written for globular length scale d_p.The various friction factors are related by

$$f_{f,bp} = \frac{f_{ker}}{3} = f_f.$$

Bird et al. [77] states that the Burke-Plummer friction factor coefficient is

$$f_{f,bp} = \frac{3.5}{6},$$

which is equals to the value given by (29) found for the capillary Fanning friction factor pressure loss model. The same scaling approach applied to the Darcy pressure loss model shows that the drag resistance coefficient for spherical particles is

$$c_{d,Dar} = f_{f,D} = \frac{8k_{kc}}{Re_{por}}, \tag{32}$$

or if following the finding by Fand et al. [80] that the Kozeny-Carman coefficient is equals $k_{kc} = 5.34$ then

$$f_{f,D} = \frac{42.72}{Re_{por}}, \tag{33}$$

as pointed out by Travkin and Catton [17]. Discussions of a number of porous media pressure resistance models can be found in analysis of different authors see, for example, Bird et al. [77], Fand et al. [80], Chhabra [78], etc.

The reason for the current description is that it shows how the transformation and comparison of correlation equations and results obtained for diverse morphology media and written with various coordinates and scaling units can be provided on the basis of a unified VAT approach. Firstly introduced before the WWII in the transport in porous media literature, the general porous media length scale $l_{por} = 4\ \langle m\rangle / S_w$ is still rarely used in various engineering circles.

Based on the above, one can make the conclusion that most experimental correlations can be reformulated for use in the SVAT bulk 1D momentum equation,

$$\frac{\Delta P}{L} = f_f\left(Re_{por}\right)\left(\frac{S_w}{\langle m\rangle}\right)\frac{\rho_f\, \tilde{\overline{U}}^2}{2}. \tag{34}$$

With this expression, any Fanning friction factor correlation can be easily compared and analyzed after reformulating the corresponding expressions in terms of Re_{por} as shown in Fig. 5. The correlations presented in Fig. 5 are for experiments carried out by the following:
1) Gortyshov et al. [84] correlations were experimentally derived for the Reynolds-Forchheimer momentum equation (porous medium resistance equation) in the form

$$\alpha = 6.61 \cdot 10^7 \left(\overline{d}_h\right)^{-1.98} < m >^{(-4.75)},$$

$$\beta = 5.16 \cdot 10^2 \left(\overline{d}_h\right)^{-1.07} < m >^{(-11.16)}, \tag{35}$$

when the hydraulic diameter $\overline{d}_h$ taken in millimeters. These correlations are for a highly porous ($< m >= 0.87 \div 0.97$) foamy metallic media.

2) A Darcy type of friction factor obtained by Gortyshov et al. [85] for very low conductivity porous porcelain with high porosity is

$$f_D\left(Re_h\right) = \frac{40}{Re_h}\left(1 + 2.5 \cdot 10^{-2} < m >^{-8.8} Re_h\right),\ < m >= 0.83 \div 0.92, \tag{36}$$

where

$$Re_h = \frac{\tilde{\overline{U}}\, d_h < m >}{\nu}.$$

To modify this equation to the SVAT momentum equation form, one needs to replace Re_h with Re_{por} and devide by 4 to obtain the Fanning friction factor

$$
\begin{aligned}
f_f\left(Re_{por}\right) &= \frac{1}{4}\left[\frac{40}{Re_{por} < m >}\left(1+2.5\cdot 10^{-2} < m >^{-8.8} Re_{por} < m >\right)\right], \\
< & \; m >= 0.83 \div 0.92, \qquad (37)
\end{aligned}
$$

to

$$Re_h \cong Re_{por} < m > .$$

3) The correlation derived by Beavers and Sparrow [86] for foam morphologies is

$$f_{bs}\left(R_w\right) = \frac{1}{R_w} + 0.074, \qquad (38)$$

with the Reynolds number suggested by Ward [87] given by

$$R_w = \frac{\widetilde{\widetilde{U}} < m > \sqrt{k_D}}{\nu},$$

is just the another problem if the permeability k_D is not known. After some algebra performed to relate this equation to the SVAT general form of pressure loss, the Fanning friction factor f_f written for the Beavers and Sparrow [86] pressure loss coefficient finally is

$$f_{f,bs}\left(Re_{por}\right) = \left(\frac{1}{Re_{por}}\left(\frac{4\sqrt{\alpha}}{S_w}\right) + 0.074\right)\left[\frac{2\sqrt{\alpha} < m >^3}{S_w}\right], \qquad (39)$$

where

$$\alpha = \frac{1}{k_D}.$$

One of experimental curves in Fig. 5 is given on the basis of the pressure loss data obtained at the University of California for the SiC foam (Fig. 6) highly regarded as one of the prospective porous media in high technology applications (Travkin and Catton [88]).