Nonlinear Effects in Multiple Regime Transport of Momentum in Longitudinal Capillary Porous Medium Morphology

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ABSTRACT

Equations for transport of momentum, heat and mass species in a regular porous media based volume averaging theory (VAT) as known to be different than in a media with irregular or random space structures. Consistent closure models are developed for these equations for media with irregularities in a substantially regular porous media. One dimensional straight parallel pore morphology (SPPM) is chosen because analytical solutions for bulk permeability and dispersion coefficients can be obtained. A single phase fluid medium is considered with the potential for accompanying transport of a dilute specie. Numerical simulation results are presented for a canonical morphology consisting of specified, stationary distributions of binary and random diameter distributions of straight pores. Unexpected results were obtained for various flow regime momentum transport in irregular media demonstrating the influence of the both weak and sufficiently large deviations in the porous medium's morphology. Suggested the Poiseuill like equation for this morphology with no adjusted but strictly defined parameters.

NOMENCLATURE

C - mass fraction concentration [-]
$\not{C}_d$ - mean skin friction coefficient over the turbulent area of MS_w [-]
c_d - mean drag resistance coefficient in the REV [-]
c_{dp} - mean form resistance coefficient in the REV [-]
c_{fL} - mean skin friction coefficient on the REV laminar region [-]
ds - interface differential area in porous medium [m^2]
d_{ch} - character pore size in the cross section [m]
D - molecular diffusion coefficient [m^2/s]
d_h - capillary morphology characteristic pore diameter [m]
d_i - diameter [m] of i-th pore [m]
d_m - mean diameter of medium's pores [m]
d_p - particle diameter [m]
Eu_{por} -porous medium Euler number [-]
f_f -Fanning friction factor [-]
g -gravitational constant [m/s^2]
h_r - pore scale microroughness layer thickness [m]

MS_w - internal surface in the REV [m^2]
K - permeability [m^2]
K_c - turbulent diffusion coefficient [m^2/s]
K_{cf} - analytic longitudinal diffusion coefficient [m^2/s]
K_e - absorption equilibrium constant [-]
K_m - turbulent eddy viscosity [m^2/s]
L_{por} - porous medium general scale [m]
+m, - averaged porosity [-]
N - number of pores [-]
n_i - number of pores with diameter of type i [-]
p - pressure [Pa]
Pe_{por} - $=\{U\}_f L_{por}/D$, porous medium Peclet number [-]
Pe_r - particle radius Peclet number [-]
Pr - $</a_f$, Prandtl number [-]
Re_{por} - $=\{U\}_f L_{por}/<$, porous medium Reynolds number [-]
Re_m - $=\{U\}_f d_m/<$, mean Reynolds number based on mean pore diameter [-]
S - cross sectional flow area [m^2]
S_c - source or sink in diffusion equation [s^{-1}]
S_{cr} - total cross sectional area available to flow [m^2]
S_{pr} - projected obstacle area on the cross perpendicular pore surface [m^2]
S_w - specific surface of a porous medium $MS_w/)S$ [1/m]
S_{wp} - $= S_z/)S$ [1/m]
S_z - cross flow projected area of obstacles [m^2]
t_T - time interval for averaging of turbulent variable [s]
u - velocity fluctuation [m/s]
U_n - dimensionless porous medium interstitial velocity [-]
U_m - characteristic velocity in porous medium, equals to laminar regime velocity in d_h tube, Eq. (32) [m/s]
u^2_{*rk} - square friction velocity at the upper boundary of h_r averaged over surface MS_w [m^2/s^2]

Subscripts

f - fluid phase
L - laminar
n - nondimensional
r - roughness
s - solid phase
T - turbulent
w - wall
i - species or pore type

Superscripts

$\hat{I}$ -value in fluid phase averaged over the REV
* -equilibrium values at the assigned surface
$^\circ$ -value in solid phase averaged over the REV
x - mean turbulent quantity

$\dot{:}$ - mean cross-section quantity in a pore

v - fluctuation value in a phase

Greek letters

$\#_C$ - averaged mass transfer coefficient over MS_w [m/s]
$)S$ - representative elementary volume (REV) [m^3]
8 - Darcy friction coefficient in tubes [-]
$:$ - dynamic viscosity [kg/(m s)] or [Pa s]
$<$ - kinematic viscosity [m^2/s]
D - density [kg/m^3]
J_w - wall shear stress [N/m^2]

INTRODUCTION

Determination of flow-variables and scalar transport for problems involving heterogeneous porous media is difficult, even when subject to simplifications allowing the specification of medium periodicity or regularity. Conventional linear or linearized models fail to properly account for transport phenomena, requiring separate dynamic coefficient models that must be solved to correct for their short-comings. Additionally, the correct form of the equations describing processes in a heterogeneous media remains an area of current debate among researchers (see Travkin and Catton, 1992). Allowing inhomogeneities to adopt random or stochastic character further confounds the already difficult task of properly identifying important transport mechanisms and predicting transport phenomena.

At present, the prevailing opinion is that "It is generally realized that it is possible to model transport properties of porous media without any reference to pore geometry, and merely use a large number of adjustable parameters in the model that do not have any physical meaning" (see Dullien, 1991). The functional dependence of porous medium characteristics like permeability on its elements like pore diameter, length, pore roughness etc. seems to be non-existent at the present time, apart from a straight pore bundle with laminar regime (Scheidegger, 1960)..

Research on porous medium morphology reveals that more and more realistic structures in the pore network image must be treated (see, for example, Mann and Yousef 1991). Inasmuch as each of the significant morphology structural elements can be randomly assigned, (Mann and Yousef, 1991; and other studies), workers have gradually developed a more sophisticated, randomized, network morphology, with up to 5 degrees of randomness; 1) pore surface roughness: 2) pore diameter; 3) pore

length; 4) pore pathway between modes (for tortuosity) and 5) pore cross-sectional shape.

In the work by Sahimi (1988), diffusion controlled reaction and transport of species is being modeled as a network of branching pores, "with no closed loops". It is called the "wood" approximation and does not allow the study of real pore networks. Another drawback of this approach is that diffusion along the pore from one node to another has been approximated by a finite difference scheme, and with constant inter-node spacing, the pore length is fixed. These are serious restrictions and they have also been noted by Kheifets and Neimark (1982).

With contemporary technology, measurements of the total void fraction, the density of the solid phase, the relative pore size distribution for pores larger than 30 angstroms in diameter, and the equilibrium adsorption characteristics of the porous medium solid phase (Sircar and Rao, 1990) can be made with confidence. There is a problem of how to connect the porous medium morphology characteristics that can and are being measured to the transport equations and coefficients.

The method of effective medium approximation (EMA) has been used by several researchers and shows some good results, see Koplik (1981). Essential differences exist in our approach and the commonly used EMA approach where the actual problem pore network is replaced by a network with artificial uniform morphology (Sahimi, 1988; Mojaradi and Sahimi, 1988; Jerauld et al. 1984; Koplik, 1981; and etc) and effective properties such as the effective diffusivity, D_{eff}, or the effective reactivity, R_{eff}. An important consideration, most often not dealt with, is determining which network morphology properties can be assigned externally, or from an effective properties point of view, which can be justified. Some of the network characteristics should be calculated to match the problem features and not assigned.

Another important point to note is that much of the reported research results are based on substitution of porous medium properties and functions using artificial network simulation capabilities without satisfying the initial equations. The concern is that the physical problem, which is simulated with the help of network modeling, must still be modeled on the basis of its initial physical and mathematical statements. The development of models of irregular and random networks of pores in the REV using closure models for the morpho-convective and morpho-diffusive terms into the transport equations is part of our current research. Numerical modeling and application of theoretical studies to this kind of closure approach is based on realities of medium morphologies.

Direct Numerical Modeling (DNM) was applied to a straight parallel pore morphology medium (SPPM) by Yuan et al. (1991) to derive a thermal dispersion coefficient, which is actually an effective one-temperature coefficient of conductivity. To obtain analytical solutions they made a number of assumptions (for example the interface surface - wall heat flux assumed to be a constant value) from which one let authors to separate the solutions of equations for initially conjugate statement.

Treating specific types of medium irregularity or randomness introduces complicated additional terms in the generalized VAT based governing equations (Whitaker, 1967, 1977, 1986a,b; Scherban et al., 1986a,b; Travkin and Catton, 1992, 1993, and Travkin et al., 1994a, Catton and Travkin, 1996). The additional terms, which are not seen in typical governing equation sets commonly used for heterogeneous media, determine the nature of the solutions and are the central issue of the closure problem. Treatment of the additional terms becomes crucial once the governing, averaged equations are written. The present study attempts to implement some basic departures from strictly regular morphology descriptions of the porous medium and illustrates a method for evaluation of some of the less tractable, additional terms present in the VAT governing equations.

There are several "morpho-fluctuation" terms in each of the VAT based equations that need to be modeled. Closure models allow one to find connections between experimental correlations for bulk processes and the present simulation representation and to incorporate them into numerical procedures. These relationships can either be analytical expressions determined with the aid of approximate theoretical evaluation or experimental correlations. As found in a study by Crapiste et al. (1986), the usual way of solving the closure problem particular to porous medium transport requires solution of a more complex integro-differential equation set for fluctuations in a real fluid volume domain. Consequently, knowledge of, or an assumption about, the porous medium morphology is required.

Methods for obtaining closure of the governing equations bulk processes in monodisperse regular and non-specified, random polydisperse media using experimental correlations have been described by Travkin and Catton (1992, 1995), Travkin et al., (1993) and Gratton et al. (1993, 1996). Travkin and Catton (1992, 1995), and Gratton et al. (1996) also compared various coefficient models by numerical simulation of flow within a two-dimensional porous layer. The current study illustrates a closure scheme for a specified capillary morphology model - straight parallel pore morphology (SPPM), by observing system behavior over an ensemble of realizations for the chosen morphology. Exact closure of the equations of flow and heat transfer are possible for this canonical morphology.

A goal of this paper is to demonstrate a general approach to modeling of transport phenomena in porous media based on volume averaging. A particularly simple geometry is chosen so that all the elements of the method can be exercised and it can be demonstrated to be complete. In this study the closures are developed for capillary medium morphology models. It is shown that the approach taken to close the integral resistance terms in the momentum equation for a regular structure can be obtained in a way that allows the Darcy-Kozeny-Carman or Forchheimer type second order terms for laminar and turbulent regimes, as shown by Travkin and Catton (1992a, 1995), to naturally appear for different flow velocities. For the case of this simple morphology, the integro-differential equation set is closed and the closure modeling approach used provides a framework for the closure process concerning problems involving more complex heterogeneous morphologies.

THEORY

SIMPLIFIED GOVERNING EQUATIONS FOR FORCED TRANSPORT IN SPECIFIC TYPES OF POROUS MEDIA

The VAT based turbulent transport equations for a single phase fluid in a highly porous random media have more integral and differential terms than in the convenient homogenized or classical continuum mechanics equations. Numerous examples are found in work by Whitaker (see, for example, 1977, 1986a,b, etc.) for laminar transport in porous media. The porous medium structural morphology determines the importance of these terms and the range of application of the various closure schemes can easily be shown to clearly manifest themselves in the order of magnitude of the predictions of both local and bulk flow parameters even for single-porous medium structural elements (e.g. the coefficient for a single bead, or single straight pore). These investigations involved the evaluation of strictly regular media morphologies, which allows the omission of many morphology-specific differential and integral terms from the general VAT based set governing equations.

The VAT based 1-D turbulent K-theory momentum equation is depicted as follows (Scherban et al., 1986b, Catton and Travkin, 1996):

$$\frac{\partial}{\partial x}\left(\langle m \rangle \left(\tilde{K}_m + \nu \right) \frac{\partial \tilde{\bar{U}}}{\partial x} \right) + \frac{\partial}{\partial x}\left(\langle \hat{K}_m \frac{\partial \hat{\bar{u}}}{\partial x} \rangle_f \right) + \frac{\partial}{\partial x}\left(\langle - \hat{\bar{u}} \hat{\bar{u}} \rangle_f \right) = <m> \tilde{\bar{U}} \frac{\partial \tilde{\bar{U}}}{\partial x} - \frac{1}{\Delta \Omega} \int_{\partial S_w} \left(K_m + \nu \right) \frac{\partial \bar{U}}{\partial x_i} \cdot \vec{ds} + \frac{1}{\rho_f \Delta \Omega} \int_{\partial S_w} \bar{p} \vec{ds} + \frac{1}{\rho_f} \frac{\partial}{\partial x} \left(<m> \tilde{\bar{p}} \right), \tag{1}$$

where the averages are defined as follows:

1. Average turbulent quantity in a pore,

$$\bar{U} = \frac{1}{t_T} \int_{t_T} U \, dt.$$

2. Average over the fluid phase within the REV,

$$\langle \bar{U} \rangle_f = \frac{1}{\Delta \Omega} \int_{\Delta \Omega_f} \bar{U} \, d\omega = \langle m \rangle \{\bar{U}\}_f = \langle m \rangle \tilde{\bar{U}}.$$

3. Average over the solid volume within the REV

$$\langle C \rangle_s = \frac{1}{\Delta \Omega} \int_{\Delta \Omega_s} C \, d\omega = \langle s \rangle \{C\}_s = \langle s \rangle \hat{C}, \; \langle s \rangle = 1 - \langle m \rangle .$$

4. Average over the entire REV,

$$\langle C \rangle = \frac{1}{\Delta\Omega} \int_{\Delta\Omega} C \, d\omega = \langle C \rangle_f + \langle C \rangle_s \, .$$

and "^" denotes the fluctuations of mean turbulent variables in the fluid domain.

Steady, fully developed momentum transport in a simplified spatial structure of straight capillaries SPPM, see Fig. 1, often used as a canonical morphological model of porous media, serves as a test problem for which exact closure to Eq. (1) can be demonstrated. For this equation, closure is achieved by modeling the overall drag resistance factor, c_d, the second and third terms on the right hand side, and second and third morpho-diffusive terms on the left hand side the equation. The very simple medium morphology allows one to obtain exact and definite closure.

The morphology of the medium shown in Fig. 1 possess the following features:
1) the spatial distribution of pores is stationary in the lateral direction and accordingly, the velocity deviations are also,
2) the second morpho-diffusive term on the left hand side (1), no longer requires retention,
3) the assumed independence of momentum transport from lateral coordinate far from the interface surface removes the first term on the left hand side and
4) the constant flow along each capillary in the REV and in each neighboring REV along of x axis determines that the differential terms to be dropped.

A fully developed flow regime, far from input - output regions, in the assumed medium eliminate the differential terms in Eq. (1). The resulting VAT based momentum equation becomes

$$\begin{aligned} &\frac{1}{\Delta\Omega} \int_{\partial S_w} \left(K_m + \nu \right) \frac{\partial \bar{U}}{\partial x_i} \cdot \vec{ds} \; - \\ &- \frac{1}{\rho_f \Delta\Omega} \int_{\partial S_w} p \, \vec{ds} \;=\; \frac{1}{\rho_f} \frac{\partial}{\partial x} \left(\langle m \rangle \tilde{p} \right) \, . \end{aligned} \tag{2}$$

A few closure equations for the morpho - terms in the VAT equations like (1) and (2) were introduced and numerically evaluated by Travkin and Catton (1992, 1993) and Gratton et al. (1993) for typical regular or nonstructured porous media.

For closure of the integral terms in the above equations, one has to integrate over the interface surface (between solid and fluid) in the REV. In the case of discrete obstacles on the wall, it is the integral over the obstacle surface. The closure scheme for skin friction resistance terms in Eq. (2) is based on

$$\frac{1}{\Delta\Omega}\int_{\partial S_{wL}} \nu \frac{\partial \bar{U}}{\partial x_i}\cdot\vec{ds} + \frac{1}{\Delta\Omega}\int_{\partial S_{wT}} K_m \frac{\partial \bar{U}}{\partial x_i}\cdot\vec{ds}$$

$$= \frac{1}{\Delta\Omega\,\rho_f}\int_{\partial S_w} (\tau_{wL}+\tau_{wT})\cdot\vec{ds} + \tag{3}$$

$$= -\frac{1}{2}(c_{fL}(x)\ S_{wL}(x) + \tilde{c}_d(x)\ S_{wT}(x))\ \tilde{\bar{U}}^2(x)\ ,$$

where interfacial stresses are represented by conventional friction coefficients for laminar and turbulent flow. For laminar subregion they are

$$\frac{\tau_{wL}}{\rho_f} = \nu\frac{\partial\,\bar{U}}{\partial\,x_i}\ , \qquad u^2_{*rk} = \frac{1}{2}c_{fL}\,\tilde{\bar{U}}^2(x)\ .$$

For turbulent flow over a rough wall, the contribution of laminar friction (index L) is usually negligible.

The integral term for the form drag resistance is closed using an analogous approach based on a pressure resistance coefficient,

$$c_{dp} = 2\frac{\int_{\partial S_w}\bar{p}\ \vec{ds}}{(\rho_f\tilde{\bar{U}}^2\ S_\perp)}\ .$$

With this coefficient, closure of the pressure resistance integral term is given by

$$\frac{1}{\rho_f\Delta\Omega}\int_{\partial S_w}\bar{p}\,\vec{ds} = \frac{1}{2}c_{dp}\ S_{wp}(x)\,\tilde{\bar{U}}^2\ . \tag{4))}$$

In the similar way can be obtained the same closed form for the 1D momentum laminar transport equations for systems with impermeable interfaces. Using the following forms of the VAT based porous media laminar momentum equation

$$0 = -\nabla \langle p \rangle_f - \frac{1}{\Delta\Omega}\int_{\partial S_w} p\,\vec{ds} + \mu\,\nabla\cdot\left(\nabla\langle m\rangle\tilde{V}\right) + \frac{\mu}{\Delta\Omega}\int_{\partial S_w}\nabla V\cdot\vec{ds} + \langle m\rangle\rho\,\vec{g}\ , \tag{5}$$

or in variable's fluctuation form

$$0 = -\langle m\rangle\nabla\tilde{p} - \frac{1}{\Delta\Omega}\int_{\partial S_w}\hat{p}\,\vec{ds} + \mu\,\nabla\cdot(\tilde{V}\nabla\langle m\rangle) + \frac{\mu}{\Delta\Omega}\int_{\partial S_w}\nabla\hat{V}\cdot\vec{ds} + \langle m\rangle\rho\,\vec{g}\ . \tag{6}$$

The later equation in 1D form is

$$\frac{\nu}{\Delta\Omega}\int_{\partial S_w}\frac{\partial\hat{U}}{\partial x_i}\cdot\vec{ds} + \nu\,\frac{\partial}{\partial x}\left(\tilde{U}\frac{\partial\langle m\rangle}{\partial x}\right) - \frac{1}{\rho_f\Delta\Omega}\int_{\partial S_w}\hat{p}\,\vec{ds} + \langle m\rangle\vec{g} = \frac{\langle m\rangle}{\rho_f}\frac{\partial}{\partial x}(\tilde{p})\ . \tag{7}$$

This equation for constant momentum flux and porosity function, with negligence of mass force can be rewritten in the form

$$-\frac{\partial}{\partial x}\{p\}_f = \frac{1}{\langle m\rangle\Delta\Omega}\int_{\partial S_w}\hat{p}\,\vec{ds} - \frac{\mu}{\langle m\rangle\Delta\Omega}\int_{\partial S_w}\frac{\partial U}{\partial x_i}\cdot\vec{ds}\ . \tag{8}$$

Meanwhile, the closure of these two integral terms can be obtained following the same procedures developed earlier by Travkin and Catton (1992) and Travkin et al. (1993) and applied above in (3), (4) for simplified turbulent momentum equation where the skin friction term is treated using conception of averaged over the REV mean skin friction coefficient c_{fL}.

Substituting these expressions into equation (8) results in

$$- \frac{\partial}{\partial x}\{p\}_f = \frac{1}{<m>\Delta\Omega}\int_{\partial S_w}(p - \tilde{p})\,\vec{ds} - \frac{\mu}{<m>\Delta\Omega}\int_{\partial S_w}\frac{\partial U}{\partial x_i}\cdot\vec{ds} =$$

$$\frac{1}{<m>\Delta\Omega}\int_{\partial S_w} p\,\vec{ds} + \frac{\tilde{p}}{<m>}(\nabla<m>) - \frac{\mu}{<m>\Delta\Omega}\int_{\partial S_w}\frac{\partial U}{\partial x_i}\cdot\vec{ds} =$$

$$\frac{1}{2}c_{fL}(x)\,\frac{S_{wL}(x)}{<m>}\,(\rho_f\tilde{U}^{\,2}(x)) + \frac{1}{2}c_{dp}\,\frac{S_{wp}(x)}{<m>}(\rho_f\tilde{U}^{\,2}(x)),$$

$$- \frac{\partial}{\partial x}\{p\}_f = \left(c_{fL}(x)\,S_w(x) + c_{dp}\,S_{wp}(x)\right)\frac{(\rho_f\tilde{U}^{\,2}(x))}{2<m>} = \tag{9}$$

$$\left(c_{fL} + c_{dp}\frac{S_{wp}}{S_w}\right)\frac{S_w}{<m>}\,\frac{\rho_f\tilde{U}^{\,2}(x)}{2} = c_d\,\frac{S_w}{<m>}\,\frac{\rho_f\tilde{U}^{\,2}(x)}{2},$$

$$c_d = \left(c_{fL} + c_{dp}\frac{S_{wp}}{S_w}\right),$$

$$c_d = \left(c_{fL}\frac{S_{wL}}{S_w} + \tilde{c}_d\,\frac{S_{wT}}{S_w} + c\right)$$

$$\frac{1}{\Delta\Omega}\int_{\partial S_w} D_f \frac{\partial \bar{C}}{\partial x_i}\cdot\vec{ds} = \frac{-1}{\Delta\Omega}\int_{\partial S_w} D_f \frac{\partial \bar{C}}{\partial n_1}\vec{n}_1\cdot\vec{ds}$$

$$= \frac{1}{\Delta\Omega}\int_{\partial S_w}\vec{q}_C\cdot\vec{ds}_1 = -\tilde{\alpha}_C S_w(\tilde{\bar{C}} - C_f^*), \qquad (13)$$

where C_f* is the equilibrium fluid phase concentration at the interface (which can be assessed through the equilibrium condition at the interface as $C_f^* = \{C\}_s K_e$), and $\vec{n}_1$ is the solid-fluid interface outward directed normal

$$\vec{ds}_1 = -\vec{ds}\,.$$

Transport Processes in SPPM. By developing procedures for velocity and concentration fluctuation calculations, the closure of the second and third terms in this equation can be found. Separate morphologies allow one to seek analytical derivations. For the classical case of parallel tube bundles with assigned non-regular or stochastic distributions of morphology characteristics, such as tube diameter and porosity, analytical expressions can be obtained for bulk characteristics, like the overall drag resistance coefficient, c_d. (10) and fluid phase dispersion coefficient. While it is sufficient for equation closure to model the overall drag resistance factor, c_d, and the morpho-diffusive and morpho-convective terms on the left hand sides of the Eq.s (11), (12), this very simple morphology is important because it allows exact and definite problem closure. The closure models developed in earlier work by Primak et al. (1986), and Shcherban et al. (1986), as well as more recent publications by Travkin and Catton (1992) and Gratton et al. (1993) for specific regular morphologies were based on experimental data for these types of porous media.

Let us represent the turbulent quantities in the averaged equations by decomposing as follows (Primak et al. 1986; Shcherban et al. 1986, Travkin and Catton 1993)

$$U = \bar{U} + u' = \bar{U}_k + \tilde{U}_r + u_k' + u_r', \qquad (14)$$

$$\bar{U} = \tilde{\bar{U}} + \hat{u}\ , \quad \hat{u} = u_r'\,,$$

where the index "k" signifies turbulence-induced components which are independent of inhomogeneities in spatial dimensions and properties resulting from the multitude of porous medium channels (pores), and "r" denotes the fluctuation contributions due to the porous medium inhomogeneity.

For straight, smooth, nonintersecting pores, the various velocity averages are represented for

the turbulent regime,

$$\hat{\hat{u}} = u_r' \neq 0, \quad \tilde{U}_r = 0, \quad \bar{U}_k = \sum_{i=1}^{N} \bar{\bar{U}}_{ki} S_i / S_{cr},$$

$$\tilde{\bar{U}} = \bar{U}_k + \tilde{U}_r = \bar{U}_k, \quad u' = u_k', \quad S_i = r_i^2 \pi,$$

$$\bar{U}_{ki} = \tilde{\bar{U}} + \hat{\hat{u}}_i = \bar{U}_k + \hat{\hat{u}}_i, \quad i = 1,....,N,$$

The velocities in the laminar regime are represented by a similar decomposition

$$U = U_k + \tilde{U}_r + u_r', \qquad (15)$$

$$U = \tilde{U} + \hat{u}, \quad \hat{u} = u_r',$$

The velocity field will be represented by these components in each of the elementary elements (cell) of the REV.

Analysis of the pore elements, both point and mean values, is best shown by considering a bi-pore morphology. The turbulent velocity in the pore of a bi-pore morphology with radius r_1 is given by

$$\bar{U}_1 = \tilde{\bar{U}} + \hat{\hat{u}}_1,$$

when the average in the REV, defined over the averaging volume $)S_f =)S_1 +)S_2$, is given by

$$\tilde{\bar{U}} = \frac{1}{\Delta \Omega_f} \left[\int_{\Delta \Omega_1} \bar{U}_1 \, d\omega + \int_{\Delta \Omega_2} \bar{U}_2 \, d\omega \right] = \frac{1}{(r_1^2 + r_2^2)} \left(r_1^2 \, \bar{\bar{U}}_{k1} + r_2^2 \, \bar{\bar{U}}_{k2} \right). \qquad (16)$$

The velocity average in the pore with radius r_1 can be related to the bi-pore average fluid velocity and the velocity fluctuation magnitude by noting that

$$\bar{\bar{U}}_{kl} = \frac{2\pi}{\pi r_1^2} \int_0^{r_1} \bar{U}_1 r\, dr = \frac{2\pi}{\pi r_1^2} \int_0^{r_1} \left(\tilde{\tilde{U}} + \hat{\hat{u}}_1 \right) r\, dr = \tag{17}$$

$$= \tilde{\tilde{U}} + a_1 .$$

where (Fig. 2)

$$a_1 = \left(\frac{2}{r_1^2} \int_0^{r_1} \hat{\hat{u}}_1 r\, dr \right) > 0.$$

Values of both fluctuation functions are shown in the Fig. 2 and it is obvious that a_2 different from a_1,

$$a_2 = \left(\frac{2}{r_2^2} \int_0^{r_2} \hat{\hat{u}}_2 r\, dr \right) < 0.$$

The average of the fluctuation function withing the REV is

$$\langle \hat{\hat{u}} \rangle_f = \frac{\langle m \rangle}{\Delta \Omega_f} \left[\int_{\Delta \Omega_1} \hat{\hat{u}}_1 d\omega + \int_{\Delta \Omega_2} \hat{\hat{u}}_2 d\omega \right] = \tag{18}$$

$$= \frac{\langle m \rangle}{(r_1^2 + r_2^2)} \left(r_1^2 a_1 + r_2^2 a_2 \right) = 0,$$

and this will be true when the number pores is arbitrarily increased.

The preceding development is for the case where the medium morphologies' (for example, in those allowing a clear analytical solution) fluctuation values can be treated as element-specific functions or constants. For the current morphology the velocity as well as coordinate and concentration field independent dispersion coefficient fluctuations can be depicted without constraint

as being element-specific. This allows one to easily restore the point-values of the fluctuation functions from a_1 and a_2 for a given SPPM, e.g. a laminar flow,

$$\hat{\bar{u}}_1(r) = 2\,\bar{\bar{U}}_{kl}\left[1 - \left(\frac{r}{r_1}\right)^2\right] - \tilde{\bar{U}} = \tilde{\bar{U}}\left[1 - 2\left(\frac{r}{r_1}\right)^2\right] + 2\,a_1\left[1 - \left(\frac{r}{r_1}\right)^2\right]. \tag{19}$$

A goal of closure of the morpho-terms in the VAT based equations is to obtain such profiles.

By limiting the scope of this work to straight, smooth, nonintersecting pores, the decomposition analytically establishes that all fluctuations are formally turbulence-induced, for the morpho-fluctuation influences vanish. Elimination of those features which happen to exist in general VAT governing equations, but are not applicable to the physical situation arising from this specific type of porous medium, reduces the equation governing the flow of fluid (9) or (2) to the two term expression

$$-\nabla \tilde{p} = c_d \left(\frac{S_w}{<m>}\right) \frac{\rho_f \tilde{\bar{U}}^2}{2} \quad , \tag{20}$$

which is pretty much the same in form to the equation routinely used for analysis of packed bed and porous media experiments. The difference is that the equation (20) is correct for only internal porous medium momentum transport and for only homogeneous constant morphology characteristics medium with strict regularity in the x-direction. The another difference is the rigor in the development of the equation from the VAT based general 1D momentum equation.

Application. The flow field in each pore is governed by well established completely defined mean value expressions

$$\bar{\bar{U}}_{ki} = \left[-\left(\frac{2\,d_i}{\lambda_i\,\rho_f}\right)\frac{d\bar{p}_i}{dx}\right]^{1/2}, \quad \lambda_i = \frac{64}{Re_{d,i}}, \quad \lambda_i = 0.3164\ Re_{d,i}^{-\frac{1}{4}}, \quad Re_{d,i} = \frac{\bar{\bar{U}}_{ki}\,d_i}{\nu},$$

with the friction resistance coefficient given for laminar and turbulent regimes respectively. In the intermediate regime where $2100 \# Re_{d,i} \# 4000$, the formulations of Kakas et al. (1987) were used to solve transcendental equations to get resistance coefficient in this range.

To assert the differentiability of random functions of the morpho-diffusive term in the momentum equation, specific conditions should be assumed for the morphology. If considering a random system of pore diameters (Fig. 1), fundamental definitions provide the basis for closing the skin friction and form drag integral terms on the right-hand side of (2) and (8), resulting in formulation of a drag coefficient sensitive to morphology characteristics and accounting exactly for skin-friction in

each individual pore,

$$c_d = \frac{\langle m \rangle}{S_w} \left[\frac{S_{cr}^2}{\left(\sum_{i=1}^{N} S_i \left(\frac{d_i}{\lambda_i} \right)^{1/2} \right)^2} \right] . \tag{21}$$

For a one diameter system, the permeability can be shown to have an exact analytical form that is both morphology and flow regime dependent,

$$K = \frac{1}{c_d \left[\frac{\tilde{\tilde{U}} S_w}{2 \nu \langle m \rangle^2} \right]} . \tag{22a}$$

Achdou and Avellaneda (1992) analyzed several microstructures to test the validity of the Johnson, Koplik and Shwartz (1986) effective medium theory in specified regimes. By considering an exactly solvable, in-parallel idealized porous microstructure consisting of nonintersecting, straight capillaries of binary diameter with circular cross-section, and furthermore assuming that all pore orientations are equally likely, simplifications for the formation factor were made and the authors

$$K = \frac{m_i^2 \left(\frac{d_{ch,i}}{2} \right)^2}{16 \langle m \rangle} , \quad i = 2 , \tag{22b}$$

established, provided that m_i is the specific volume of 2nd size capillaries and

$$\left(\frac{d_{ch,2}}{d_{ch,1}} \right) \gg 1 .$$

An analytical longitudinal diffusion coefficient for the bulk may be defined considering the same system of straight, smooth, non-intersecting circular capillaries. Introducing expressions for the longitudinal dispersion in each porous morphology element requires that various flow regimes be considered. For turbulent flow within a single pore, Taylor (1954) assumed a "universal" velocity profile, the validity of Reynold's analogy, and that the coefficients of longitudinal and lateral diffusion are equivalent,

$$K_{c,i} \equiv K_{c,i_T} = 10.1 \frac{d_i}{2} \bar{\bar{U}}_{ki} \left(\frac{\lambda_i}{8} \right)^{1/2}, \quad i = 1, \ldots, N . \tag{23}$$

It should be noted, that the classical analogy used by Taylor (1954) is actually incorrect, as the new evidences appear regarding insufficiencies of some very well adopted correlating equations (see, for example, Churchill and Chan (1995a,b), and Churchill (1997). So far, for the theory demonstration goal of present work this has no significant impact. The equation (23) is valid provided that the flow in pores of type "i" is turbulent. However, for a given, imposed pressure gradient, flow may be either laminar, or intermediate, or turbulent in the considered pore, depending on various geometric factors, thus, an effective longitudinal diffusion coefficient can be defined for the laminar pore-flow regime. Following Carbonell (1979), the effective laminar diffusion coefficient in each pore based upon the bulk, flow cross-sectional area averaged velocity is given by,

$$K_{c,i} \equiv K_{c,i_L} = D_f + \frac{\tilde{\bar{U}} \left(\frac{d_i}{2} \right)^4}{48 D_f} \left[\frac{\sum_{j=1}^{N} S_j}{\sum_{j=1}^{N} S_j \left(\frac{d_j}{2} \right)^2} \right] . \tag{24}$$

Finally, an analytical longitudinal bulk diffusion coefficient is obtained,

$$\tilde{K}_{cf} = \frac{\sum_{i=1}^{N} S_i K_{c,i}}{\sum_{i=1}^{N} S_i} . \tag{25}$$

The semi-analytical and analytical deviations can now be calculated from the following expressions

$$a_i = \bar{\bar{U}}_{ki} - \tilde{\bar{U}} , \quad \hat{K}_{ci}^{(s)} = K_{c,i} - \tilde{K}_{cf} , \tag{26}$$

where the superscript (s) means deviations per medium element (per pore). Evaluation of concentration or temperature deviations is limited. The velocity and bulk longitudinal dispersivity coefficient deviations are not dependent on the longitudinal coordinate in a morphology of straight pores with assigned in present work conditions per element. This eliminates the influence of the morpho-convective and morpho-diffusive terms in the transport equations.

It is widely accepted that when a media composed of globular morphologies can be described in terms of S_w, <m>, and d_p, for example a spherical particle medium with

$$S_w = \frac{6\,(1 - \langle m \rangle)}{d_p}, \quad d_h = \frac{2}{3}\,\frac{\langle m \rangle}{(1 - \langle m \rangle)}\,d_p,$$

the same dependency on the equivalent pore diameter results as one finds for a one diameter capillary morphology, see below, leading to

$$S_w = \frac{6\,(1 - \langle m \rangle)}{d_p} = 6\,(1 - \langle m \rangle) \Big/ \left(\frac{3}{2}\,\frac{(1 - \langle m \rangle)}{\langle m \rangle}\,d_h \right) = \frac{4 \langle m \rangle}{d_h}\ .$$

The latter dependency is a strong argument for a simple "universal" porous media length scale defined by

$$d_h = \frac{4 \langle m \rangle}{S_w}\ . \tag{27}$$

This scale meets the needs and internal logic of both major morphologies - capillary and globular, and incorporates two properties of the media, both void fraction and specific surface area. It has a solid theoretical basis for at least for two types of canonical porous media morphology (straight capillary parallel pore morphology - SPPM and one diameter sphere globular morphology) and was arrived at with different theoretical reasonings earlier by others, e.g. Kays and London (1984), etc. There are a number of analysis and experimental studies of different porous media morphology leading to the use of a porous media Reynolds number that is of the form

$$Re_{por} = \frac{4\,\tilde{\tilde{U}}\,\langle m \rangle}{\nu\,S_w}\ . \tag{28}$$

Although various porous media pressure resistance models are described by others (see, Bird et al. (1976), Fand et al. (1987), and Chhabra (1993)), the current description for scale d_h allows the transformation and comparison of correlation equations and experimental results obtained for diverse morphology media and the use of various scaling. It also allows experimentally determined characteristics of the media to be related to the closure relationships derived from the VAT based

equations by Travkin and Catton (1997a,b).

It was shown by Travkin and Catton (1992, 1995) that a good approximation for a homogeneous porous media is

$$c_d \cong f_f \,, \tag{29}$$

where f_f is the Fanning friction factor. This allows one to determine the drag resistance in Eq. (20) from the bulk value of the pressure loss coefficient, f_f, obtained from experimental correlations, when it is used as shown,

$$-\nabla \tilde{p} = f_f \left(\frac{S_w}{<m>} \right) \frac{\rho_f \tilde{\tilde{U}}^2}{2} \quad . \tag{30}$$

There are a few reasons why the media drag resistance and the Fanning friction factor are not identical. One of them is because the media inflow and outflow pressure losses are usually incorporated into the correlations for f_f in the data reduction process. Others are the result of the additional morphology - dependent terms in the general momentum VAT equation.

Having established the relationship with a conventional experimental reduction technique, one can reformulate any experimental correlation for use in the simplified VAT based bulk 1D momentum equation,

$$-\nabla \tilde{p} = c_d \left(Re_{por} \right) \left(\frac{S_w}{<m>} \right) \frac{\rho_f \tilde{\tilde{U}}^2}{2} \,. \tag{31}$$

It can be shown that the conventional two-term drag resistance coefficient expression used in the Reynolds-Forchheimer equation pressure loss equation is convertible to the Fanning factor type coefficient c_d in the (31) (Travkin and Catton, 1995, 1997b).

With this expression, any correlation for the Fanning friction factor can easily be compared and analyzed. The scales chosen for using with Eq. (31) are

$$x_m = L_{por} = \frac{4 <m>}{S_w} \,, \quad U_m = \left[\frac{\Delta p}{L_{por}} \frac{1}{8 \mu} \left(\frac{d_h}{2} \right)^2 \right], \; p_m = \Delta p \,, \tag{32}$$

where U_m is the laminar regime velocity in a tube with d_h diameter. So, the dimensionless VAT based momentum equation becomes (this implies that the pressure drop causes in a distance L_{por})

$$Eu_{por} = c_d(Eu_{por})U_n^2, \quad with \quad U_n = \frac{\tilde{\tilde{U}}}{U_m}, \tag{33}$$

where the Euler porosity number is given by

$$Eu_{por} = \left(\frac{\Delta p}{2\ \rho_f U_m^2} \right) = \frac{32\,\mu^2\, S_w^2}{\rho_f \Delta p <m>^2}\ . \tag{34}$$

With the scaling velocity given by (32), the Eq. (33) for a one-diameter SPPM will have drag resistance c_d equal to Euler number in the steady-state regime. As soon as the current SPPM allows the direct analytical evaluation, this equation may be written in the form which supply the direct solution for $c_d(Eu_{por})$ as well as for U_n.

The VAT momentum equation (34) can be written also in terms of overall discharge as an analogous equation to Poiseuill's law for straight tube

$$\frac{\Delta p}{L} = \frac{32\ \mu\ U_m}{d_h^2}\ .$$

The 1D VAT simplified nondimension momentum equation for homogeneous porous medium yields the form

$$\begin{aligned} -\frac{dp_n}{dx_n} &= c_d(Eu_{por}) \left[\frac{\rho_f <m>^2 \Delta p}{32\,\mu^2 S_w^{\,2}} \right] U_n^2 = \\ &= c_d(Eu_{por})\,(Eu_{por})^{-1}\ U_n^2\ , \end{aligned} \tag{35}$$

where $c_d(Eu_{por})$ is determined in (21). This equation is valid for every flow regime occuring in arbitrary selected pore or in an assigned distribution of pore bundle. If laminar regime occurs in each of the pore in pore bundle, it is known that this kind of momentum equation resolves with the exact analytical formulae for the drag resistance. In other regimes this equation includes the drag resistance estimations

or correlations obtained experimentally or analytically for each pore.

RESULTS AND DISCUSSION

Two types of morphology are addressed. First, a two diameter capillary morphology is studied. This is followed by a study of a capillary morphology with random diameter sizes.

Binary Set of Pores. Numerical methods were employed to calculate the coefficients described in Eqs.(21), (22), (25) and (26). Though solution of the equations does not require any special or elaborate numerical schemes, when considering a variety of morphological parameter combinations the proliferation of data quickly makes data management tedious. Hence, computations were performed and organized on a digital computer. Morphological parameters which were varied while modeling the binary, smooth, nonintersecting straight circular pores included $\langle m \rangle$, n_1, n_2 ,$d_{ch,2}$, and S_w while the small-pore diameter, $d_{ch,1}$, the total number of pores, n, and the total cross-sectional area were held constant at specified values. Results are provided for four distinct sets of morphological characteristics involving three distinct porosities. Two separate morphological characteristics corresponding to an identical porosity are given to illustrate the effects associated with altering the real, geometric structure, though certain non-dimensional and fractional quantities are equivalent.

Predictions of the drag coefficient as a function of the bulk Reynolds number are shown in Fig. 3. The drag coefficient generally decreases for increasing porosity at equivalent Re_m and the profiles all approach a horizontal asymptote as a "fully turbulent" regime is approached for a prescribed morphology. Additionally, the effect of high porosity is to advance the onset of the turbulent transition to smaller bulk Reynolds numbers. The $\langle m \rangle = 0.399$ profile with large-pore fraction, n_2/n, equivalent to 0.56 shows an appreciable reduction in c_d for $\langle m \rangle = 0.602$ and $\langle m \rangle = 0.399$ profiles, which is a coincidence. The ability of the $\langle m \rangle = 0.399$, $n_2/n_1 = 0.56$ profile to surpass the $\langle m \rangle = 0.602$ profile in performance is largely due to the lower porosity curve having a specific surface, S_w, only slightly over two-thirds that of the $\langle m \rangle = 0.602$ profile. Because no tortuosity has been introduced into the morphology model, flow resistance is provided only by wall shear. Hence, the parameter S_w exerts great influence upon the bulk flow field by the mechanism of fluid-friction. It is evident that the proper combination of $\langle m \rangle$ and S_w will greatly change the prescribed morphology flow admittance and that, furthermore, minimizing S_w can result in drastic coefficient-value improvement which may even out-pace the adverse influence of decreased $\langle m \rangle$.

Observation of the bulk permeability behavior, K, as function of the overall drag coefficient, c_d, provides useful insights into hydrodynamic regimes in which one may wish to operate, given the presumed morphology. Figure 4 graphically provides such information. General trends again show that it is desirable to maximize $\langle m \rangle$ and minimize S_w. Additionally, the presence of what appears to be noise superimposed upon the ascending portions of the curve has actually been determined to be a hysteresis structure, though captured with poor resolution. The hysteresis results from K not mapping one-to-one onto c_d in the regime surrounding transition to turbulence. If greater numerical resolution were used, the presence of upper, lower, and middle curves would be evident for each morphology characterization, corresponding to turbulent, laminar and transition regimes, respectively. The upper curve, not visible on the graph, appears as a linear extension of the high c_d asymptote back into the

transition c_d range. The lower curve is clearly evident, being the curve upon which the tooth-like structures are superimposed in Fig. 4. Finally, the middle curve, again not visible, has a slope less than that of the lower curve, but greater than the constant-valued upper curve, and lies within an envelope between the two on the graph throughout the transition c_d range. The presence of the hysteresis structure suggests that maximum benefit is derived by operating just beyond the transition to turbulence for the prescribed morphology.

Though flow conditions are inextricably tied to the morphology characteristics, they are not exclusively dependent upon these characteristics. Failure to properly account for flow regime may introduce large errors in the coefficient predictions. Figures 5 and 6 show calculations of the bulk permeability, K, resulting from the use of flow regime independence [Eq.(22b)] and the formal definition [Eq.(22a)] formulas, respectively. For values of Re_m clearly within the bulk laminar regime, agreement between Eqs.(22a) and (22b) is approximately order-of-magnitude. Recall that Eq.(22b) is of asymptotic form, requiring large ratios of $d_{ch,2}$ to $d_{ch,1}$ for accuracy. However, the data chosen for discussion has diameter ratios of O(1). The agreement between Eqs.(22a) and (22b) in the laminar regime improves greatly for larger diameter ratios. Large disagreement between the formulae arise late in the laminar regime, where the flow-regime independent formulae loses their predictive validity, with a power law increase in the disagreement between the expressions through bulk transition and throughout the bulk turbulent regimes. Use of the analytical expressions [Eq.(22b)], or flow-regime independent expressions in general, will result in gross error for regimes where Stoke's flow is not occurring in each of the pores. The permeability data presented here, for both Eqs.(22a) and (22b), are for admittedly large diameters, better mimicking artificial media than naturally occurring materials. The purpose of the study, however, is to identify and reproduce specific physical phenomena and behavior.

The dimensionless longitudinal diffusion coefficient (25) further illustrates the resulting transport coefficient enhancement derived from minimizing S_w for a given value of the bulk porosity, see Fig. 7, as was evidenced for c_d. The profile corresponding to a large-pore fraction of 0.56 and $\langle m \rangle = 0.399$ exceeds the performance of both the $\langle m \rangle = 0.602$ and the remaining $\langle m \rangle = 0.399$ profiles for equivalent Re_m. The results shown in Fig. 4 are for the bulk laminar regime.

Upon re-examination of Eq.(11), it can be seen that for a continuous distribution over a certain range of pore sizes, the longitudinal behavior of the velocity deviations and effective diffusivity deviations will have no change in the longitudinal direction while a conservation of the solute is adopted, at least for incompressible fluids. This leads to the decreasing influence of the second and third derivative expressions with respect to the longitudinal coordinate, included on the left hand side of the equation. The result is a smaller effective longitudinal coefficient than that obtained from Eq. (25), and shown in Fig. 6, because only the conventional diffusion coefficient in the diffusion term in the transport equation is accounted for. Additionally, as previously mentioned, the metrics are exaggerated. Because a component of Eq. (24) translates like $d_{ch,i}^4$, parameter overestimation, as compared to more consistent treatments, may occur. However, the behavior observed for this classical morphological model is derived from an analytically exact treatment.

Fluctuation values of the dimensionless longitudinal diffusion coefficient, $\$K_\sigma$ are shown for two different diameter-ratios, d_2/d_1, in Figs. 8 and 9. It is apparent that the inclusion of only a few pores of the larger diameter creates appreciable deviation in the parameter values. The results for the smaller pore diameter-ratio (Fig. 8) indicate a transition in the dominating influence with varying flow regime. At low Re_1, the longitudinal diffusion coefficient fluctuation value is clearly greater in the

pores of larger diameter. With increasing Re_1, $\$K_c$ values in both the small and large pores approach zero. With even greater Re_1, the fluctuation values in the small pores dominate as the fluctuation magnitudes increase for pores of both types. The described evolution is evidenced by the changing concavity of the curves shown in Fig. 8. Increased pore diameter-ratio (Fig. 9) results in a change in the low Re_1 behavior. The coefficient fluctuation values are consistently dominated by the small pores, even at low Re_1. Again, increasing Re_1 accompanies increasing fluctuation value magnitudes. Differences among the smaller and larger pore diameter-ratio curves emanate from the various flow regimes potentially realized in pores of different types (e.g. laminar flow in the smaller pores, turbulent flow in the larger pores).

Velocity fluctuation values, a_i are given in Fig. 10. As is expected, the larger pores consistently have larger velocity fluctuation values for an imposed pressure gradient. No distinct change in behavior is observed at low Re_1, as was the case with $\$K_c$, though fluctuation magnitudes do increase with Re_1. Increased d_2/d_1 serves to amplify the fluctuation values, but does not change the character of the parameter behavior. Once again, it can be seen that the introduction of only a small number of large diameter pores can drastically change fluctuation values.

The current study has examined the most fundamental morphology models by excluding consideration of pore network inter-connectivity, pore tortuosity, non-circular pore cross-sectional geometries, non-percolating passages, pore roughness, multi-scale phenomena and various other complexities which realistically mimic naturally occurring heterogeneous media. However, even this basic treatment is capable of extracting pertinent physical behavior and quite accurately models some engineered porous media.

By varying the morphological descriptor values in space in either a prescribed or random manner, the resulting fluctuations in the flow variables may be ascertained. The accuracy of the simulation will depend on the limits placed on descriptor values and the manner in which they are allowed to vary, both subsequently dependent on the explicit porous material under study. Literally, an infinite number of realizations of the medium morphology, even for the simple model presented here, are obtainable. However, the morphology and metrics of the materials of typical engineering interest are generally well known (in a statistical sense) or are obtainable with some experimental effort. Hence, correct descriptions can be identified and, subsequently, the desired transport parameters can be obtained. Additionally, a useful consequence of the current method is the ability to solve the converse problem. Given desired transport properties, a partial description of the necessary morphology may be obtained.

RANDOM DISTRIBUTION PORE SIZE MORPHOLOGY. The results of numerical simulation of specific morphology closure statements described above for straight parallel pores with random distribution are quite different qualitatively from the binary systems. One of the primary goals of this simulation was to obtain momentum transport results which are exact solutions and for which exact averaging procedures could be accomplished for a broad spectrum of flow regimes and pores diameters. Results were generated for a specified range of diameters then compiled using an appropriate distribution to guide the sampling. The distributions of pore radii were taken as uniform and normal. The results from our study of a binary set of pores were used as benchmark results for comparison with the results of stochastic pore distribution analysis and their verification.

The first series of calculations was made for a diameter range of 0.0 to 0.003 m including fine pores and pores with diameters of 0.001 m order of magnitude. Fine pores include pores with

diameters of 10^{-6} m and less. They were modeled for different flow regimes in different pores and for various assigned pressure drops. A laminar flow regime in each pore was achieved by choosing a low enough pressure drop. Nevertheless, even for a uniform flow regime and uniform pore radius distribution, outcome functions sometimes reflect an extreme shift toward the left side of the result ranges, see Figs. 11 through 15. The drag coefficient magnitudes are distributed over seven orders of magnitude, while the pore diameter extends only over three orders of magnitude.

Results for another range of pore diameters (d_i = [0.003, 0.006] m) yielded velocity distributions (Fig. 16) that bear significant resemblance to the initial diameter distribution only when in the same laminar flow regime. The drag resistance distribution shifts toward the larger values. One can reach the conclusion that when a given range of pore size and initial physical characteristics have some uniformity, the medium morphological characteristics determine the transport process characteristics. This is true for only specific situations. Increasing the Eu_{por}^{-1} number made some pores rendered as in the intermediate and turbulent regimes with changed distribution of the velocity values, Fig. 17.

Increasing the pore diameter interval range increases the breadth of the spectrum of results. The drag resistance distribution for a pore diameter range of d_i = [0, 0.03] m shows the extreme shift to the left side of the velocity p.d.f. pattern, see Fig. 18. Even more evident is the extremely low (relatively) drag resistance in most of the pores. This means that large pore momentum transport dominates. Meanwhile the same right shifting pattern is recognizable when the assigned pressure drop is increased (as is the Eu_{por}^{-1} number) and a bimodal distribution evident, see Fig. 19.

The velocity distribution in an artificially wide pore morphology, (d_i = [0, 0.06] m) explicitly showed also two modes. Meanwhile, the drag resistance coefficient distribution demonstrates an extreme left shift of the p.d.f., toward very low values due to high velocity rate in the larger pores with transitional and turbulent regimes of flow. The same shift to the left is seen for the mass dispersion coefficient distribution. The velocity distribution for this pore diameter range has the same clear bimodal behavior as the velocity p.d.f. shown in Fig.19.

Fig. 20 summarizes the resistance coefficient (Fanning friction factor) data for few SPPM pore distributions:

a) uniform distribution in range (0,3]mm;

b) bi-pore morphologies with different numbers of second diameter pores, $n_2 = 10^3$, 10^4, and 10^5, d_2 = 3mm, n_1 = 1057534 ($d_1 = 3 \text{Å} 10^{-4}$ m);

c) normal pore diameter distribution with the mean diameter d_{mean} = 3/2mm and standard deviation F = 1mm.

An experimental correlation for the drag resistance coefficient c_d for SiC ceramic foam with porosity <m> = 0.756 and specific surface S_w = 8058.6[1/m] obtained at the University of California, Los Angeles (Travkin and Catton, 1997b) is also given for comparison. One of obvious observations from Fig. 20 is that in the laminar regime all the coefficient curves regardless of diameter distribution mode lay lower then the Fanning friction factor law for a single tube (pore).

Written in terms of ($\{K_c\}_f$ - Pe_{por}), as done in many studies see, for example, Carbonell (1979), Levec and Carbonell (1985), and others, the results of present work (see Fig. 21) show substantial disagreement with the results of other authors. While the comparison in Fig. 22 reveals 2 to 4 orders of magnitude difference between this work and analytical general formulae developed by, for example, Saffman (1959) and others, one needs to realize that compared are different dispersivity coefficients while for quite different morphologies. Saffman (1959, 1960) based his model on a

random walk by fluid particles through the lattice of randomly oriented straight pores of a diameter such that " each step corresponding to passage through one pore, of variable direction and duration". Even such an artificial construction gave an approximate agreement when additional rules where adopted, as pore length l being " put equal to the average diameter of the particles composing the porous medium". The only morphological parameters participating in the final expression for longitudinal and lateral dispersion coefficients were pore length l and radius a. The type of dispersivity concerned in present work is one that valid in only fluid phase and in straight parallel pore medium morphologies.

Yuan et al. (1991) estimated the thermal dispersion coefficient for a similar morphology, and included longitudinal thermal dispersion in the solid phase. The dispersion coefficient was written for a combined one temperature SPPM model, but the analysis was done using a two-temperature model for the flow direction temperature distribution in both the straight pores and in the surrounding solid. Including the solid phase heat transport greatly effected the magnitude of the overall thermal dispersion coefficient resulting in its being reduced in magnitude sufficiently enough to compare with various experimental results. Yuan et al. (1991) reached the conclusion that -"Under certain circumstances, heterogeneities are predicted to cause several orders of magnitude increase in the thermal dispersion coefficient". The current work yielded a similar conclusion.

Another interesting point is that declining values of $\{K_c\}_f$ at large $Pe_{por} > 10^5$ can be explained by the much lower turbulent dispersivity in a single straight pore (Taylor, 1953, 1954) than when a laminar regime with a lower Re numbers occurs in the same pore. No other correlation could predict this effect.

CONCLUSIONS

Justification of the choice of calculation formulae used to determine flow and mass transfer parameters for a porous media is both important and difficult. In most of the existing techniques for determining heat and mass transfer parameters, linear equations and constant coefficients are used while the superposition of errors due to experiment and intrinsic to the calculational formulae is ignored. Closure of mathematical models of diffusion processes in a heterogeneous media may result in the use of a quasi-homogeneous method in complex situations. In this method, the transfer process is modeled as an ideal continuum with effective transport characteristics instead of the real heterogeneous character of the porous medium. In either case, the reliability of the coefficients largely determines the reliability of the results. In the present study, the governing equations are obtained by a hierarchical modeling methodology, accompanied by an advanced averaging technique, developed and described in earlier work. The numerical work is derived from a capillary type morphology description at the pore scale. The elementary macrovolume morphologies considered are systems of straight-equivalent channels with anisotropic features and random morphology fluctuations. The random morphology fluctuations are incorporated into the transport equations by means of randomly varying morpho-convective and morpho-diffusive terms.

Further simplifications of the governing equations and the chosen morphology allow results to be obtained that show that the method has advantages over simple, deterministic Darcy law based treatments with constant coefficients. The parameter S_w exerts great influence upon the bulk flow field, through the drag coefficient, by the mechanism of fluid-friction. The simple preliminary treatments

are incapable of accounting for random variations of the solid medium and result in large magnitude error if coefficients are not chosen to correspond to the bulk flow regime and morphology of the real physical situation being modeled.

The momentum and scalar transport equations involve additional terms which quantify the influence of the medium irregularity. Theoretical forms of these additional terms are reviewed for both one- and two-dimensional cases in a porous medium morphology consisting of specified, stationary distributions of a binary and random systems of straight, non-intersecting pores. Important physical behavior is extracted from the morphology model. It is shown that slight manipulations of the morphology descriptors can create large fluctuations in transport parameter values, signifying the potential modeling errors incurred if particular features of the morphology are neglected. Hydrodynamic flow regime considerations also significantly effect the transport parameter values. A method for closure of the morpho-fluctuation terms in the governing transport equations is outlined. The method allows either the determination of expected transport parameters given a description of the porous medium, or conversely, qualities of the porous medium given description of the desired transport parameters. The objective of this modeling was to determine the impact of a random pore distribution on averaged and fluctuation values of momentum and mass dispersion coefficient distributions.

Exact modeling in each straight pore was combined using theoretical closure analysis for a given morphology. Two kinds of pore diameter distributions were used: 1) two-mode regularly distributed pore diameters; 2) uniformly distributed diameters in the range 0 to a few millimeters or few centimeters and 3) normally distributed diameters.

These distributions resulted in dramatically different distributions of velocity, dispersion coefficient and other characteristics. An analysis of statistical data modeled on exact solutions for assigned capillary morphology was provided. It showed the tremendous importance of the lowest size pore distribution phenomena. It was found that a negligible part of the overall momentum transport goes through the smallest pores, yet the heat and mass transport through these pores is a large fraction of the bulk transport due to their significant interface transport and enhanced transport peculiarities in fine pores.

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