A Two-Temperature Model for Turbulent Flow and Heat Transfer in a Porous Layer

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ABSTRACT

A new model of turbulent flow and of two-temperature heat transfer in a highly porous medium is evaluated numerically for a layer of regular packed particles. The layer can have heat exchange from the defining surfaces. The commonly used models of variable morphology functions for porosity and specific surface were used to obtain comparisons with other works in a relatevly high Reynolds number range. A few outstanding features of the closure models for additional integral terms in equations of flow and heat transfer are advanced. Closures were developed for capillary and globular medium morphology models. It is shown that the approach taken to close the integral resistance terms in the momentum equation for a regular structure can be obtained in a way that allows the second order terms for laminar and turbulent regimes to naturally occur. These terms are taken to be close to the Darcy term or Forchheimer terms for different flow velocities. The two-temperature model was compared with a one-temperature model using thermal diffusivity coefficients and effective coefficients from various authors. Calculated pressure drop along a layer showed very good agreement with experiment for a porous structure of spherical beads. A simplified model with constant coefficients was compared with analytical solutions.

Nomenclature

a	-	thermal diffusivity [m^2/s]
A_4	-	morphology similarity number [-]
b	-	mean turbulent fluctuation energy [m^2/s^2] $= \frac{1}{2}\overline{u'_i u'_i}$
B_1	-	similarity number in turbulent kinetic energy equation [-]
$\tilde{c}_d$	-	mean skin friction coefficient over the turbulent area of ∂S_w [-]
c_d	-	mean drag resistance coefficient in the REV [-]
c_{dm}	-	drag resistance scale and similarity number [-]
c_{dp}	-	mean form resistance coefficient in the REV [-]
c_{fL}	-	mean skin friction coefficient over the laminar region inside of the REV [-]
c_f^N	-	local skin friction coefficient [-]
c_p	-	specific heat [J/(kg K)]
C_1	-	constant coefficient in Kolmogorov turbulent exchange coefficient correlation [-]

Da	-	Darcy number [-] = K/H^2
d_{ch}	-	character pore size in the cross section [m]
d_p	-	particle diameter [m]
dS	-	interphase differential area in porous medium [m^2]
MS_w	-	internal surface in the REV [m^2]
$\tilde{f}$	-	averaged over) S_f value f
$<f>_f$	-	value f, averaged over) S_f in a REV
$\hat{f}$	-	value f morpho-fluctuation in a) S_f
g	-	gravitational constant [$1/m^2$]
H	-	width of the channel [m]
h	-	half-width of the channel [m]
h_r	-	pore scale microroughness layer thickness [m]
K	-	permeability [m^2]
K_b	-	turbulent kinetic energy exchange coefficient [m^2/s]
k_f	-	fluid thermal conductivity [W/(m K)]
$k_{f,e}$	-	effective thermal conductivity of fluid [(W/(m K)]
k_m	-	stagnant effective conductivity of porous medium [W/(m K)]
k_s	-	solid phase thermal conductivity [W/(m K)]
$\tilde{K}_m$	-	averaged turbulent eddy viscosity [m^2/s]
K_{ST}	-	effective thermal conductivity of solid phase [W/(m K)]
K_T	-	turbulent eddy thermal conductivity [W/(m K)]
K_w	-	similarity number in eddy viscosity boundary condition [-]
l	-	turbulence mixing length [m]
L	-	scale [m]
m	-	porosity [-]
¢ m ¦	-	averaged porosity [-]
m_0	-	mean porosity [-]
Nu_w	-	Nusselt number on the external wall [-]
Nu_z	-	Nusselt number across the porous layer [-]
p	-	pressure [Pa] and pitch in regular porous 2D and 3D medium [m]
Pe	-	local Peclet number [-] = VL/a
Pr	-	Prandtl number [-] =$</a_f$
Pr_T	-	turbulent Prandtl number [-] =K_m/K_T
Q_0	-	outward heat flux [W/m^2]
Q_{w2}	-	similarity number in the boundary conditions for temperatures [-]
Re_{ch}	-	Reynolds number of pore hydraulic diameter [-]
Re_p	-	particle Reynolds number [-] =$\tilde{u}\, d_p/<$
S_w	-	specific surface of a porous medium $MS_w/)S$ [1/m]
S_{wm}	-	characteristic scale for a specific surface [1/m]
S_{wp}	-	[1/m] =$S_z/)S$
S_{w0}	-	mean specific surface [1/m]
S_z	-	cross flow projected area of obstacles [m^2]
T	-	temperature [K]
T_a	-	characteristic temperature for given temperature range [K]
T_m	-	convective fluid temperature scale in the porous layer [K]
T_{fm}	-	mean convective fluid temperature across the porous layer [K]
T_s	-	averaged over) S_s temperature [K]
T_w	-	wall temperature [K]
T_0	-	reference temperature [K]

u	-	velocity in x-direction [m/s]
u_0	-	mean velocity in the layer [m/s]
u_4	-	velocity outside of the momentum boundary layer (at the center of the porous layer) [m/s]
u^2_{*rk}	-	square friction velocity at the upper boundary of h_r averaged over surface MS_w [m^2/s^2]
V	-	velocity [m/s]
V_D	-	Darcy velocity [m/s]
w	-	velocity in z-direction [m/s]

Subscripts

e	-	effective
f	-	fluid phase
i	-	component of turbulent vector variable
k	-	component of turbulent variable that designates turbulent "microeffects" on a pore level
L	-	laminar
m	-	scale value
r	-	roughness
s	-	solid phase
T	-	turbulent

Superscripts

Î	-	value in fluid phase averaged over the REV
x	-	mean turbulent quantity
N	-	turbulent fluctuation value
*	-	nondimensional value

Greek letters

$\#_T$	-	averaged heat transfer coefficient over MS_w [W/(m^2K)]
$\#_{T,m}$	-	mean heat transfer coefficient across the layer [W/(m^2K)]
$"_{Tm}$	-	characteristic heat transfer coefficient scale [W/(m^2K)]
$"_w$	-	heat transfer coefficient at the wall [W/(m^2K)]
)S	-	representative elementary volume (REV) [m^3]
$)S_f$	-	pore volume in a REV [m_3]
$)S_s$	-	solid phase volume in a REV [m_3]
F_b	-	turbulent coefficient exchange ratio K_m/K_b [-]
F_T	-	turbulent coefficient exchange ratio K_m/K_T [-]
8	-	friction coefficient in tubes [-]
:	-	dynamic viscosity [Pa s]
<	-	kinematic viscosity [m^2/s]
D	-	density [kg/m^3]
D_0	-	reference density [kg/m^3]
J	-	turbulent friction stress tensor [N/m^2] $= -\rho\overline{u'w'}$
J_w	-	wall shear stress [N/m^2]

1 Introduction

Most non-Darcian studies are based on a model summarized by the equation

$$-\frac{1}{\rho_f}\frac{\partial p}{\partial x} = \frac{\nu}{K}V_D + b\,V_D^{\,2}, \tag{1}$$

where b is a constant, determined either experimentally or analytically, and U is the Darcian velocity. In recent years many studies have been performed using the Brinkman-Forchheimer momentum equation models (David et al., 1991; Kladias and Prasad, 1990; Georgiadis and Catton, 1987a; and others). David et al., (1991), using a porosity function $m(\vec{x})$, write

$$\frac{m\,\mu\,V_D}{K} + \frac{m\,\rho_f F\,|V_D|\,V_D}{\sqrt{K}} = -\nabla p_D + \mu\nabla^2 V_D - \beta\rho_0(T - T_0)\,m g, \tag{2}$$

where

$$p_D = m(\vec{x})\,p\,, \quad V_D = m(\vec{x})\,V,$$

whereas Kladias and Prasad (1990) allege that the same governing momentum equation should be

$$\frac{\rho_f}{m^2}(V_D\cdot\nabla)V_D = -\nabla\tilde{p} - \frac{\mu}{K}V_D - \frac{\rho_f b}{K}\,|V_D|\,V_D + \mu'\nabla^2 V_D + \rho_f g, \tag{3}$$

where

$$\mu' \neq \mu\,.$$

$$\begin{aligned}(V\cdot\nabla)\,m\,V = &-\frac{m}{\rho_f}\nabla p + \frac{\mu}{\rho_f}\nabla^2(m\,V)\\ &-\frac{\mu}{\rho_f K}V - g\,m\,\beta(T - T_0),\end{aligned} \tag{4}$$

Excluding discrepancies connected with the free convection term, we see that some other terms also differ. Only in the most recent literature has the nonuniformity of the porosity function been treated by keeping it within differential signs. In Hayes' (1990) work, we find

$$\nabla(mV) = 0,$$

$$m\,V\cdot\nabla T = \nabla(a_e\nabla T)\,.$$

Hayes refers to an article by Du Plessis and Masliyah (1988), where a "new mathematical model is proposed for time-independent laminar flow through a rigid isotropic and consolidated porous medium of spatially varying porosity." Despite the importance of transition and clear turbulence motion in porous media, not much work could be found. In work by Ward (1964), experimental results based on a Fanning friction factor were obtained,

$$f_k = \frac{dp}{dl} \frac{\sqrt{K}}{V^2 \rho_f} = \frac{1}{R_k} + 0.55 \ , \tag{5}$$

where the Reynolds number for porous media is defined as

$$R_k = \frac{V K^{0.5}}{\nu} .$$

It can be seen that f_k 6 0.55 for large R_k. The view exists that one may use equation (2) or (3) without the quadratic term while modeling a dual porous structure. This choice has no rigorous substantiation.

If the energy equation is written with the assumption that, the fluid and solid are homogenized, as is done in most past work, then

$$(\rho c_p)_f V_D \nabla T = \nabla (k_m \nabla T). \tag{6}$$

If the energy equation contains the effective thermal diffusivity a_e, then it has two components

$$a_e = a_n + a_d,$$

where

$$a_n = \begin{cases} a_m - \text{stagnant coefficient, or} \\ a_0 - \text{molecular diffusivity ,} \end{cases}$$

$$a_d - \text{underestimated coefficient.}$$

Experimental investigation of heat transfer from a wall with the constant temperature in highly porous media $0.94 <= m_0 <= 0.97$, was performed by Hunt and Tien (1988a,b). Nield (1991) expressed some doubts about the terms in the final form of the averaged momentum equation. Slow laminar flow and heat transfer through a porous flat channel with isothermal boundaries were considered in the research of Kaviany (1985). The solution of the equations used by Kaviany, close to those used by Vafai and Tien (1981), showed the influence a porous medium morphology parameter ($= (h^2 m_0/K)^{1/2}$. In the work by Vafai and Thiyagaraja (1987) the effects of flow and heat transfer near an interface region between two porous media, porous medium and fluid region and near solid wall were investigated employing the governing equations with constant porosities. Influence of three parameters on heat and laminar momentum transport in two-dimensional porous medium have been studied in the work by Vafai and Sozen (1990). Namely there were parameters of particle Reynolds number Re_p, Darcy number Da, and the ratio of the solid to vapor phase diffusivities. There was found that Da number is the most important factor in determining the assumption of local thermal equilibrium.

There is a noticeable lack of experimental measurements of the real characteristics inside the porous medium. The absence of appropriate experimental methods has been the explanation. A promising article was recently published by Georgiadis et al. (1991). The article contains very interesting figures with the averaged structure, shown in small scale details, of porosity and velocity functions across a tube containing a packed bed. Unfortunately, the paper does not contain the needed details of the experiment such as the local Reynolds number, the distribution of specific surface, and descriptions of the porous medium inner channels, which could be, and most likely will be established by exploiting the experimental technique.

It is our view that more attention should be focused on the theoretical development needed to understand turbulent flow and heat and mass transfer exchange in the channels of complex configurations with various wall roughness, and consequently, on well described structures of porous media, since there are very few physically substantiated and fully developed theories.

2 Development of Turbulent Transport Models in Highly Porous Media

Let us assume that the obstacles in a globular porous medium structure and the channels between them are arranged randomly. A regular arrangement of obstacles is far more easily studied than a randomly nonhomogeneous one. The former has all of the merits of a canonical model which is suitable both for comparison with experiment and exact solutions as well as for treating conducting boundary transitions in the course of simulation of a process.

Derivation of the equations of turbulent flow and diffusion for a highly porous medium during the filtration mode is based on the theory of averaging over a certain REV (representative elementary volume) $\Delta\Omega$ (Whitaker, 1967) of the turbulent transfer equation in the fluid phase and the transfer equations in the solid phase of a heterogeneous medium (Scherban et al., 1986). Modeling of heterogeneous (two energy equation) non-Darcian flow for transient operation of a particle bed has not, thus far, been fully investigated. Experimental studies have revealed that at high Peclet numbers, transient operations in packed beds cannot be modeled correctly with a single energy equation for the medium. The turbulent transport statement set was considered here for the case of turbulent flow in a highly porous layer (Fig. 1) combined with a two energy equation model. The transfer equations for each of the phases are used with some simplification, determined by the turbulence order and medium regularness (Travkin and Catton, 1992a).

The REV in the porous layer is defined as the volume contained in a plane rectangular region parallel to the subsurface or a disk whose horizontal dimensions are far greater than characteristic dimensions of obstacles and infinite in one transverse direction. In the vertical direction perpendicular to the subsurface, the REV thickness is considerably smaller than that in the horizontal directions. The structure of selected REV forms is dictated by features of the highly porous layer medium and, specifically, by the length of its scales in different directions. Similar modeling assumptions about the macrovolume orthotropy, in the course of averaging, have been made (Whitaker 1967, Immich 1980) on the basis of specific features by posing the problems from a physical standpoint. The equations for turbulent boundary transfer in a porous layer under steady-state conditions and with penetration through an interface which were derived by Scherban et al. (1986) and augmented by Travkin and Catton (1992a,b,c) can be simplified to the following (see Appendix A)

$$\frac{\partial}{\partial x}(\langle m\rangle \tilde{\tilde{u}}) + \frac{\partial}{\partial z}(\langle m\rangle \tilde{\tilde{w}}) + \frac{1}{\Delta\Omega}\int_{\partial S_w}(\tilde{\tilde{u}}_i + \hat{\tilde{u}}_i)\cdot \vec{dS} = 0,$$

$$\langle m\rangle \tilde{\tilde{u}}\frac{\partial \tilde{\tilde{u}}}{\partial x} + \langle m\rangle \tilde{\tilde{w}}\frac{\partial \tilde{\tilde{u}}}{\partial z} - \frac{\tilde{\tilde{u}}}{\Delta\Omega}\int_{\partial S_w}(\tilde{\tilde{u}}_i + \hat{\tilde{u}}_i)\cdot \vec{dS} =$$

$$-\frac{1}{\rho}\frac{\partial}{\partial x}(\langle m\rangle \tilde{\tilde{p}}) + \frac{\partial}{\partial z}\left[\langle m\rangle\left(\overline{-u_k^{\prime} w_k^{\prime}}\right)\right] + \langle m\rangle \tilde{F}_i - \tag{7}$$

$$\frac{\partial}{\partial z}(\langle \hat{\tilde{u}}\hat{\tilde{w}}\rangle_f) - \frac{1}{\Delta\Omega}\int_{\partial S_w}\bar{u}_j\bar{u}_i\cdot \vec{dS} - \frac{1}{\rho\Delta\Omega}\int_{\partial S_w}\bar{p}\,\vec{dS} -$$

$$\frac{1}{\Delta\Omega}\int_{\partial S_w}\overline{u_{jk}^{\prime}u_{ik}^{\prime}}\cdot \vec{dS}, \qquad j = 1,3.$$

The term with $\tilde{F}_i$ will specifically reflect the impact of microroughness and augment the previous level of the simulation hierarchy. The importance of this roughness has been shown by many authors. A one-dimensional model, under steady-state conditions and other simplifications, with no flow penetration through ∂S_w, for a horizontally homogeneous stream, has the form

$$\frac{\partial}{\partial z}\left(\langle m\rangle\left(\tilde{K}_m+\nu\right)\frac{\partial\tilde{\bar{u}}}{\partial z}\right)+\frac{\partial}{\partial z}\left(\langle\hat{K}_m\frac{\partial\hat{\bar{u}}}{\partial z}\rangle_f\right)+$$

$$+\frac{\partial}{\partial z}\left(\langle-\hat{\bar{u}}\hat{\bar{w}}\rangle_f\right)+\frac{1}{\Delta\Omega}\int_{\partial S_{wT}}K_m\frac{\partial\bar{u}}{\partial x_i}\cdot d\vec{S}+ \tag{8}$$

$$+\frac{1}{\Delta\Omega}\int_{\partial S_{wL}}\nu\frac{\partial\bar{u}}{\partial x_i}\cdot d\vec{S}-\frac{1}{\rho_f\Delta\Omega}\int_{\partial S_w}\bar{p}\,d\vec{S}=\frac{1}{\rho_f}\frac{\partial\langle\bar{p}\rangle_f}{\partial x}.$$

An equation for turbulent flow in a porous layer with regular packed bed structure characteristics for the steady-state case with impermeable interphase surface is simplified to

$$\frac{\partial}{\partial z}\left[<m(z)>\left(\tilde{K}_m(z)+\nu\right)\frac{\partial\tilde{\bar{u}}(z)}{\partial z}\right]+\frac{1}{\Delta\Omega}\int_{\partial S_{wT}}K_m\frac{\partial\bar{u}}{\partial x_i}\cdot d\vec{S}$$
$$+\frac{1}{\Delta\Omega}\int_{\partial S_{wL}}\nu\frac{\partial\bar{u}}{\partial x_i}\cdot d\vec{S}-\frac{1}{\rho_f\Delta\Omega}\int_{\partial S_w}\bar{p}\,d\vec{S}=\frac{1}{\rho_f}\frac{\partial(<m(z)>\tilde{\bar{p}})}{\partial x}. \tag{9}$$

To take the influence of volume drag resistance forces of the porous media on the turbulent fluctuation energy balance into consideration, one must consider the contribution of pulse character drag forces. Assuming that most of the mean motion kinetic energy lost due to interaction of the flow with the porous medium solid obstacles translates into increasing of the turbulent fluctuation energy (Monin and Yaglom, 1975; Menzhulin, 1970) will bring one to the conclusion that

$$\overline{X'\,u'}=c_d S_w u^3.$$

It follows that the equation for the mean turbulent fluctuation energy b(z) can be written in the form

$$\tilde{K}_m(z)\left(\frac{\partial\tilde{\bar{u}}}{\partial z}\right)^2+\frac{d}{dz}\left(\left(\frac{\tilde{K}_m}{\sigma_b}+\nu\right)\frac{db(z)}{dz}\right)+\frac{f_1(c_d)\,S_w(z)}{<m>}\tilde{\bar{u}}^3-$$
$$-2\nu\left(\frac{db^{1/2}(z)}{dz}\right)^2-\frac{g}{T_a\sigma_T}\left[\tilde{K}_m\frac{\partial\tilde{\bar{T}}}{\partial z}\right]=C_1\frac{b^2(z)}{\tilde{K}_m}, \tag{10}$$

where $f_1(c_d)$ is approximately that given by the equation for constant and nearly constant morphology functions and the mean eddy viscosity is given by

$$\tilde{K}_m(z)=C_1^{1/4}\,l(z)\,b^{1/2}(z), \tag{11}$$

where l(z) is the turbulent scale function defined by the assumed porous medium structure.

In the same way, with all of the previous simplifications, the equation of turbulent heat transfer in the homogeneous porous medium fluid phase is

$$c_{pf}\rho_f<m>\tilde{\bar{u}}(z)\frac{\partial\tilde{\bar{T}}(x,z)}{\partial x}=\frac{\partial}{\partial z}\left[<m>\left(\tilde{K}_T+k_f\right)\frac{\partial\tilde{\bar{T}}(x,z)}{\partial z}\right]$$
$$+\frac{1}{\Delta\Omega}\int_{\partial S_{wT}}K_T\frac{\partial\bar{T}}{\partial x_i}\cdot d\vec{S}+\frac{1}{\Delta\Omega}\int_{\partial S_{wL}}k_f\frac{\partial\bar{T}}{\partial x_i}\cdot d\vec{S}, \tag{12}$$

and in the solid phase

$$\frac{\partial}{\partial z}\left[(1 - <m(z)>)\, K_{sT}(z)\frac{\partial T_s(x,z)}{\partial z}\right] + \frac{1}{\Delta\Omega}\int_{\partial S_w} K_{sT}\frac{\partial T_s}{\partial x_i}\cdot \vec{dS_1} = 0\,, \tag{13}$$

where

$$\vec{dS_1} = -\vec{dS}\,.$$

3 Development of Closure Models for Some Regular Capillary and Globular Kinds of Porous Media

There exist two main approaches to the development of closure models for non-Darcian or turbulent two-phase transport equations in highly porous media. It is implied that these equations are properly derived, yet they are now in doubt. The first approach is to develop an appropriate turbulence model. The second is to treat the numerous integrated and covariant terms in averaged transport equations.

To deal with uncertainties in determining and modelling effective coefficients in each phase, more fundamental developments are required. Among the problems that remain beyond the development of quantitative approaches, is, more importantly, the development of a general approach that can account for real differences between the structural and physical properties of each of the porous medium components. In most cases, researchers have not clearly described the simplifying assumptions they used to obtain the averaged equations and transport coefficient models. For example, models for transport coefficients are derived without any real consideration of the medium morphology or the introduction of a different averaging technique. According to a common and well-known approach, the simplest porous medium morphological models are the following

A. Capillary (porous) models:
   1. A bundle of parallel tubes , all of the same diameter, d.
   2. A bundle of parallel slits , all of the same width, h, embedded in a solid, including numerous variations of the parallel tube and slit model.

B. Globular morphology models:
   1. A set of one size beads (spheres) uniformly or nonuniformly placed in a fluid -- this model is known as the "dusty-gas" model.
   2. A sets of parallel tubes, having definite distributions of diameter and spatial parameters.

All of the foregoing morphology simplifications have been exploited for laminar fluid flow (see, for example Kheifets and Neimark, 1982; Dullien, 1992 and other's successive works). Similar models can be used for turbulent flow in porous media and for diffusion models when fluctuations are ignored (these developments see by Shvidler 1986, 1989 for the laminar regime). The natural way to close the integral terms in the transfer equations is to attempt to integrate over the interphase surface, or of some other outlined areas of this surface. The skin friction resistance terms in equation (9) are

$$\begin{aligned}
&\frac{1}{\Delta\Omega}\int_{\partial S_{wL}} \nu\,\frac{\partial \bar{u}}{\partial x_i}\cdot\vec{dS} + \frac{1}{\Delta\Omega}\int_{\partial S_{wT}} K_m\frac{\partial \bar{u}}{\partial x_i}\cdot\vec{dS}\\
&\quad = \frac{1}{\Delta\Omega\,\rho_f}\int_{\partial S_w}(\tau_{wL}+\tau_{wT})\cdot\vec{dS}\\
&= -\frac{1}{2}\left(c_{fL}(z)\;S_{wL}(z) + \tilde{c}_d(z)\,S_{wT}(z)\right)\tilde{\bar{u}}^{\,2}(z)\;,\\
&\frac{\tau_{wL}}{\rho_f} = \nu\,\frac{\partial\,\bar{u}}{\partial\,x_i}\,,\qquad u^2_{*rk} = \frac{1}{2}c_{fL}\tilde{\bar{u}}^{\,2}(z)\;.
\end{aligned} \tag{14}$$

The pressure drag resistance integral term is closed in a manner similar to that for a one component pressure resistance coefficient over a single obstacle,

$$c_{dp} = 2 \frac{\int_{\partial S_w} \bar{p} \, d\vec{S}}{(\rho \, \tilde{\bar{u}}^{2} \, S_{\perp})} . \tag{15}$$

The result is

$$\frac{1}{\rho_f \Delta\Omega} \int_{\partial S_w} \bar{p} \, d\vec{S} = \frac{1}{2} c_{dp} \, S_{wp}(z) \, \tilde{\bar{u}}^{2} . \tag{16}$$

Closure of the heat exchange integral terms, based on similar relationships, is

$$\begin{aligned} &\frac{1}{\Delta\Omega} \int_{\partial S_w} K_T \frac{\partial \bar{T}}{\partial x_i} \cdot d\vec{S} = \frac{-1}{\Delta\Omega} \int_{\partial S_w} K_T \frac{\partial \bar{T}}{\partial n_1} d\vec{S} \cdot \vec{n}_1 \\ &= \frac{1}{\Delta\Omega} \int_{\partial S_w} \vec{q}_T \cdot d\vec{S}_1 = - \tilde{\alpha}_T \, S_w \, (\tilde{\bar{T}} - T_s) , \end{aligned} \tag{17}$$

where S_w = MS$_w$/) S. The vector $\vec{n}_1$ is the outward directed normal to the solid phase. The final set of equations includes (10), (11) and, using the closure models described above, the equation of motion

$$\begin{aligned} &\frac{\partial}{\partial z} \left[<m(z)> \tilde{K}_m (z, \tilde{\bar{u}}, b, l) \frac{\partial \tilde{\bar{u}}(z)}{\partial z} \right] \\ &= \frac{1}{2} \left[c_{fL}(z, \tilde{\bar{u}}) S_{wL}(z) + \tilde{c}_d (z, \tilde{\bar{u}}) S_{wT}(z) + c_{dp}(z, \tilde{\bar{u}}) S_{wp}(z) \right] \tilde{\bar{u}}^{2} \\ &\qquad + \frac{1}{\rho_f} \frac{d<\bar{p}>_f}{dx} , \end{aligned} \tag{18}$$

where $\tilde{K}_m = \tilde{K}_m + \nu$,

the energy equation in fluid phase

$$\begin{aligned} &c_{pf} \, \rho_f <m> \tilde{\bar{u}}(z) \frac{\partial \tilde{\bar{T}}(x,z)}{\partial x} \\ &= \frac{\partial}{\partial z} \left[<m> \tilde{K}_T(z) \frac{\partial \tilde{\bar{T}}(x,z)}{\partial z} \right] + \\ &\tilde{\alpha}_T(z) \, S_w(z) \, (T_s(x,z) - \tilde{\bar{T}}(x,z)) , \end{aligned} \tag{19}$$

with (x,z) 0) S$_f$, and the energy equation in the solid phase

$$\frac{\partial}{\partial z}\left[(1 - <m(z)>) K_{ST}(z) \frac{\partial T_s(x,z)}{\partial z}\right]$$
$$= \tilde{\alpha}_T(z)\, S_w(z)\, (T_s(x,z) - \tilde{\bar{T}}(x,z)), \quad (x,z) \in \Delta\Omega_s\,, \tag{20}$$

with

$$Pr_T \cong 1: \quad \tilde{K}_T \cong \tilde{K}_m c_{pf} \rho_f + k_f\,.$$

In the equations (10), (11), (18)-(20), the coefficient functions and specific surface functions must be determined by assumed real or invented morphological models of the porous structure. The pressure gradient term in equation (18) is modeled as a constant value in the layer, or simulated by the local value of the right hand side of the experimental correlations. The boundary

$$z = 0: \quad \tilde{\bar{u}} = 0\,, \frac{\partial b}{\partial z} = 0\,, \; \tilde{K}_m = \nu\,, \; Q_0 = -\tilde{K}_T \frac{\partial \tilde{\bar{T}}}{\partial z}\,, \; Q_0 = -K_{ST} \frac{\partial T_s}{\partial z}\,,$$
$$z = h: \quad \frac{\partial \tilde{\bar{u}}}{\partial z} = 0\,, \; \frac{\partial b}{\partial z} = 0\,, \; \frac{\partial \tilde{\bar{T}}}{\partial z} = 0\,, \; \frac{\partial T_s}{\partial z} = 0\,. \tag{21}$$

conditions for these equations are the following

The fundamental distinction between the present resistance model and those exploited before is the second power of velocity in all additive terms and for either laminar or turbulent regimes. The closure model is simple and presents a clear physical problem statement as well as incorporating three additional terms. The velocity power difference can be determined by another initial approach and consequently another initial set of equations along with the natural closure procedures considering a realistic assumption of the morphology structure of porous medium. In section 4, it will be shown that most experimental correlations can be related to the present work. Watanabe (1989) obtained a similar second power result for pressure drop dependence for granular packed beds when he used the drag force law for a single isolated sphere to derive the pressure drop formula for beds. It is an example of an attempt to accurately take into account the assumed morphology of a porous medium while evaluating it's numerical characteristics.

4 Transport Coefficients for Definite Kinds of Medium Morphology

In recent years, determination of effective transport coefficients has received considerable attention by researchers who are studying the transport of heat (or a conservative solute) through fully-saturated porous media by advection and diffusion. Georgiadis and Catton (1986) showed that the Forchheimer-Brinkman extended model could explain the divergence of the Nusselt versus Rayleigh number data in terms of a new parameter which depends on the porous medium Prandtl number and the ratio d/L. The "effective" thermal conductivity of the medium was identified with its "stagnant" value obtained under no-flow conditions. In view of certain discrepancies between predicted and measured Nusselt number values for the porous Bénard problem, Georgiadis and Catton (1987b) revised the constant conductivity model. They used a single energy equation with an "effective" thermal conductivity expressed as a linear function of the local Pe number

$$k = k_m + c\,Pe\,. \tag{22}$$

Here it should be noted that linearly combining the second term with the stagnant mix thermal conductivity is an oversimplification of the physics.

In considering a porous medium canonically formed from tubes and slits, we will review some existing theoretical derivations and unify the various points of view and conclusions. One type of porous media will be considered with its various mathematical incarnations for the flat porous layer momentum equation. Momentum equation given by Vafai and Kim (1989),

$$\frac{\partial^2 (<m> \tilde{u})}{<m>\; \partial z^2} = \frac{<m>}{K} \tilde{u} + \frac{<m>^3}{\nu} \frac{F}{K^{1/2}} \tilde{u}^2 + \frac{1}{\mu} \frac{d\tilde{p}}{dx}\,, \tag{23}$$

Poulikakos and Renken (1987)

$$\frac{\partial^2 \tilde{u}}{\partial z^2} = \frac{1}{K(z)}\tilde{u} + \frac{A(z)}{\nu}\tilde{u}^2 + \frac{1}{\nu \rho_f}\frac{d\tilde{p}}{dx}, \tag{24}$$

and by this work

$$\frac{\partial}{\partial z}\left[<m>\frac{\partial \tilde{u}}{\partial z}\right] = \frac{1}{2\nu}\left[c_{fL}S_w + c_{dp}S_{wp}\right]\tilde{u}^2 + \frac{1}{\nu\rho_f}\frac{d<p>_f}{dx}, \tag{25}$$

$$<p>_f = <m>\tilde{p},$$

for the laminar regime momentum equation are considered. Further, drag resistance relationships derived from these equations for the middle part of the channel where the porosity is constant are, Vafai and Kim (1989) equation

$$-\frac{d\tilde{p}}{dx} = \frac{\mu}{K(z)}<m>\tilde{u} + \rho_f <m(z)>^3 \frac{F}{K(z)^{1/2}}\tilde{u}^2, \tag{26}$$

Poulikakos and Renken (1987) equation

$$-\frac{d\tilde{p}}{dx} = \frac{\mu}{K(z)}\tilde{u} + \rho_f A(z)\tilde{u}^2, \tag{27}$$

as well as the Ergun's (1952) relation

$$\frac{\Delta p}{L} = -\frac{d\tilde{p}}{dx} = \frac{\mu}{K(z)}<m(z)>\tilde{u} + \rho_f A(z)<m(z)>^2\tilde{u}^2, \tag{28}$$

and that obtained from this work for the laminar regime (turbulent or combined regime c_d should be calculated accordingly to the closure schemes)

$$\begin{aligned} -\frac{d\tilde{p}}{dx} &= \left[c_{fL} + c_{dp}\frac{S_{wp}}{S_w}\right]\frac{S_w(z)}{<m(z)>}\left(\frac{\rho_f\tilde{u}^2}{2}\right) \\ &= c_d\frac{S_w(z)}{<m(z)>}\left(\frac{\rho_f\tilde{u}^2}{2}\right), \end{aligned} \tag{29}$$

where

$$c_d = \left[c_{fL} + c_{dp}\frac{S_{wp}}{S_w}\right],$$

is drag resistance coefficient in a porous medium. These correlations of drag resistance can be evaluated for packed beds by noting that the last expression is the experimental relationship for pressure drop in packed beds

$$\frac{\Delta p}{L} = f\left(\frac{S_w}{<m>}\right)\left(\frac{\rho_f\tilde{u}^2}{2}\right). \tag{30}$$

It can be seen that a good approximation for nearly-constant or large-scale nonuniform porosity is given by

$$c_d - f, \tag{31}$$

where f is taken from experimental correlations. It is easy to see that this approximation for resistance forces is restricted by variability of the morphology functions m(x), S_w and others. Ergun's relation in terms of the previous formulae will be

$$\frac{\Delta p}{L} = f_{ER} \left(\frac{S_w}{<m>} \right) \frac{\rho_f \tilde{\tilde{u}}^2}{2}, \tag{32}$$

where

$$f_{ER} = \frac{A}{Re_{ch}} + B, \quad A = 33.33, \quad B = 0.583,$$

and for spherical particles

$$Re_{ch} = \frac{2}{3} \frac{<m>}{(1 - <m>)} \frac{\tilde{\tilde{u}} d_p}{\nu}.$$

These parameters are very close to those of the Klenov and Matros (1990) correlation with A_{ch} =36.3 and B_{ch} =0.45 for spherical particles and A_{ch}=37.6 and B_{ch} =0.585 for cylindrical particles. The formula for drag resistance of a spherical particle bed derived from Watanabe (1989) results in a drag resistance coefficient that corresponds to the present notation

$$f_w = c_{sph}\, 25(1 - <m>)/12 = c_{d,w}, \tag{33}$$

where $c_{\underline{sph}}$ is the drag resistance coefficient of a single isolated bead of diameter d_p . The drag resistance coefficient derived from the correlations of Fand and Thinakaran (1990), for a large range of Reynolds numbers

$$10^{-5} < Re_{sph} = d_p u/< < .\ 0(10^3),$$

is

$$c_{d,ft} = f_{ft} = 2A/(9Re_{ch}) + B/3, \tag{34}$$

where A and B are given by Fand and Thinakaran (1990). The drag resistance coefficient for spherical particles in the Darcy regime is

$$c_{dar} = f_{dar} = 42.72/Re_{ch}. \tag{35}$$

These relationships will be used in the next set of correlations. The ideal parallel tubes morphology model yields the following coefficient models

$$\lambda = \frac{8\tau_w}{\left(\rho_f \tilde{\tilde{u}}^2\right)}, \quad \tau_w = \frac{d_{ch}\Delta p}{4\Delta L},$$

$$u_*^2 = \frac{\tau_w}{\rho_f} = \frac{\lambda \tilde{\tilde{u}}^2}{8}, \quad \frac{\Delta p}{\rho_f \Delta L} = -\frac{d\tilde{\tilde{p}}}{\rho_f dx} = \frac{2}{R} u_*^2. \tag{36}$$

A straight equal diameter tube morphology model yields the morphology function S_w/<m>,

$$S_w = \frac{\partial S_w}{\Delta\Omega} = \frac{2\pi R}{p\Delta y}, \quad <m> = \frac{\pi R^2}{p\Delta y}, \quad \frac{S_w}{<m>} = \frac{2}{R}. \tag{37}$$

and there results

$$\frac{\Delta p}{L} = \lambda \frac{1}{2R}\frac{\rho_f \tilde{\bar{u}}^2}{2}, \tag{38}$$

from this

$$\frac{\Delta p}{L} = \frac{\lambda}{4}\left(\frac{2}{R}\right)\frac{\rho_f \tilde{\bar{u}}^2}{2} = \frac{\lambda}{4}\left(\frac{S_w}{<m>}\right)\left(\frac{\rho_f \tilde{\bar{u}}^2}{2}\right),$$

and one obtains

$$c_d = \tilde{c}_d \cong \frac{\lambda}{4}.$$

The friction coefficient $\tilde{c}_d$ for smooth tubes calculated with the Nikuradze and Blasius formulae is

$$c_d = \tilde{c}_d = \frac{\lambda}{4} = \frac{\left(0.0032 + \frac{0.221}{Re^{0.237}}\right)}{4},$$
$$c_d = \tilde{c}_d = \frac{\lambda}{4} = \frac{\left(0.3164\, Re^{-0.25}\right)}{4}. \tag{39}$$

For straight pipes with uniform sand roughness, a model representing the porous morphology is that given by the Nikuradze formula for the friction coefficient in turbulent flow

$$c_d = \tilde{c}_d = \frac{\lambda_r}{4} = \frac{\left(\frac{1}{(0.87\ \ln(R_{ch}/h_r)} + 1.74\right)^2}{4}. \tag{40}$$

Here R_{ch} is the effective radius of a pore. Evaluation of the friction coefficient in tubes with two-dimensional roughness of different forms was made by Kader (1977). A model representing slit morphology of a porous medium was treated in conformity with definition

$$\tau_w = \frac{\Delta P}{\Delta L} h = c_f^{\prime} \frac{\rho\, \tilde{\bar{u}}^2}{2}, \quad c_f^{\prime} = \frac{2\,\tau_w}{\rho\, \tilde{\bar{u}}^2} = \frac{2\, u_*^2}{\tilde{\bar{u}}^2}. \tag{41}$$

The morphology ratio $S_w/<m>$ for a porous medium morphology model of straight equal slits is

$$S_w = \frac{(2L\Delta y)}{(pL\Delta y)} = \frac{2}{p}, <m> = \frac{(HL\Delta y)}{(pL\Delta y)} = \frac{H}{p}, \frac{S_w}{<m>} = \frac{2}{H} = \frac{1}{h}, \tag{42}$$

which yields

$$-\frac{d\tilde{p}}{dx} = \frac{\Delta p}{L} = c_f\left(\frac{1}{h}\right)\left(\frac{\rho_f \tilde{\bar{u}}^2}{2}\right),$$

where

$$c_d = \tilde{c}_d \cong c_f.$$

The data for numerically verifying the calculated drag resistance coefficient c_{dp} and combined (approximately) coefficient $(c_{dp} + \tilde{c}_d)$ for a rough slit morphology model were taken from the paper of Taylor et al. (1985)

$$\log c_{dp} = -0.125 \log(Re_d) + 0.375 , \qquad Re_d = \tilde{u} \frac{d_r}{\nu} < 6 \cdot 10^4 , \tag{43}$$

where d_r is the diameter of a single projected roughness element on the pore wall and

$$c_{dp} = 0.6, \qquad Re_d \geq 6 \cdot 10^4 , \qquad c_d = (c_{dp} S_{\perp} / \partial S_w + \tilde{c}_d) \equiv (0.003 \div 0.015) . \tag{44}$$

To evaluate the heat transfer coefficient using various known methods it is helpful to start with the analysis done in the review papers of, for example, Kaguei et al. (1983). Kokorev et al. (1987) established an interesting correlation between drag coefficient and heat transfer coefficient for turbulent flow in porous medium that contains only one empirical (apparently universal) constant. On the basis of this relationship, a concept of fluctuation speed scale of movement, is used to determine an expression for the heat transfer coefficient

$$\tilde{\alpha}_T = \frac{k_f}{d_p} \left(0.14 \, (c_d \, Re_{ch}{}^3)^{1/4} \, Pr^{1/3} \right). \tag{45}$$

The many heat trancfer expressions given in the Heat Exchanger Design Handbook (1983) are based on the use of a single sphere heat transfer coefficient for the porous medium, as is done here,

$$\tilde{\alpha}_T = \frac{k_f}{d_p} \left(f_{\psi} \, Nu_s \right), \qquad Nu_s = 2 + (Nu^2{}_l + Nu^2{}_T)^{1/2} , \tag{46}$$

where

$$Nu_l = 0.664 \, Re_p^{1/2} \, Pr^{1/3} , \qquad Re_p = \frac{\tilde{u} \, d_p}{\nu} ,$$

$$Nu_T = \frac{(0.037 \, Re_p^{0.8} \, Pr)}{(1 + 2.443 \, Re_p^{-0.1} \, (Pr^{2/3} - 1))} ,$$

for $1 < Re_p < 10^5$, $0.6 < Pr < 10^5$, and with the form coefficient for $0.26 < \langle m \rangle < 1$, equals

$$f_{\psi} = 1 + 1.5 (1 - \langle m \rangle) .$$

Direct simulation of variable morphology functions for regular spherical beads packing is explained in a paper by Gratton et al. (1993) and Travkin and Catton (1994a).

5 Numerical Method of Equation Solution

The governing equations are cast into dimensionless form using the following scales

$$z_m = x_m = \frac{m_0}{S_{wm}}\,,\; u_m = \left(-\frac{z_m}{\rho_f}\,\frac{d\,\tilde{p}}{d\,x} \right)^{1/2},\; b_m = {u_m}^2\,,\; T_m = \frac{z_m\,Q_0}{K_{Tm}}\,,$$

$$K_{mm} = z_m\,u_m\,,\; K_{Tm} = \quad K_{mm}\,c_{pf}\,\rho_f\,,\;\; S_{wm} = \frac{6\,(\,1 - m_0\,)}{d_p} \vee S_{w0}\,,\;\; m_0\,, \tag{47}$$

$$\alpha_{Tm} = \frac{K_{Tm}}{z_m}\,,\quad c_{dm} = \frac{2\,{u_m}^2}{{u_0}^2}\,.$$

The scaled governing equations are

$$\frac{\partial}{\partial z^*}\left[<m^*>K_m^*\,\frac{\partial u^*}{\partial z^*}\right] \;=\; \frac{1}{2}\,c_{dm}\;c_d^*\;S_w^*\;u^{*2} \;-\; A_4\;, \tag{48}$$

where

$$A_4 = \frac{1}{m_0}\,,$$

$$\begin{aligned} &K_m^*\left(\frac{\partial u^*}{\partial z^*}\right)^2 \;+\; \frac{d}{dz^*}\left(K_b^*\,\frac{db^*}{dz^*}\right) \;+\; c_{dm}\left(\frac{c_d^*\,S_w^*}{<m^*>}\right)u^{*^3} \;- \\ &\qquad 2\,K_w\left(\frac{d\,b^{*\,1/2}}{d\,z^*}\right)^2 \;-\; B_1 K_m^*\,\frac{\partial\,T_f^*}{\partial\,z^*} \;=\; C_1\,\frac{b^{*^2}}{K_m^*}\,, \end{aligned} \tag{49}$$

where

$${K_m}^* \;=\; C_1^{1/4}\,l(z^*)\,b^{*^{1/2}}\,, \tag{50}$$

$$K_w \;=\; \frac{\nu}{z_m\,u_m}\,,\quad B_1 \;=\; \frac{T_m\,z_m\;g}{u_m^2\,T_a\,\sigma_T}\,,\quad {K_b}^* \;=\; \left(\frac{{K_m}^*}{\sigma_b} \;+\; \frac{\nu}{z_m\,u_m}\right),$$

$$\begin{aligned} &<m^*>u^*(z^*)\,\frac{\partial\,T_f^*(x^*,z^*)}{\partial\,x^*} \;= \\ &= \;\frac{\partial}{\partial z^*}\left[K_T^*<m^*>\frac{\partial\,T_f^*}{\partial\,z^*}\right] \;+\; \alpha_t^*\,S_w^*\left[\,T_s^*(x^*,z^*) \;-\; T_f^*(x^*,z^*)\,\right], \end{aligned} \tag{51}$$

where

$$<m^*> \;=\; <m^*(z^*)>,\quad \alpha_T^* \;=\; \alpha_T^*(u^*,m^*,S_w^*),\quad S_w^* \;=\; S_w^*\,(z^*)\,,$$

and

$$\frac{\partial}{\partial z^*}\left[(A_4 - <m^*>) K_{ST}^* \frac{\partial T_s^*}{\partial z^*} \right] = \alpha_T^* S_w^* [T_s^* - T_f^*] . \tag{52}$$

The scaled boundary conditions are

$$K_m^* \Big|_{z = +0} = K_w ,$$
$$- K_T^* \frac{\partial T^*}{\partial z^*} \Big|_{z = +0} = Q_{w2} , \qquad Q_{w2} = \frac{Q_o}{c_{pf} \rho_f u_m T_m} . \tag{53}$$

Both the scaling and the thermal transfer boundary conditions influence the general model equation structure. The same similarity number appears in the boundary conditions for the second temperature T_s^*. It is apparent that the main similarity numbers influencing the flow regime solution are the drag resistance similarity number c_{dm} and morphology number A_4 . The value of the mixing length calculated using the porous medium "effective" pore scale was taken

$$l(z) = f_l\left(\frac{4<m(z)>}{S_w(z)} \right) , \quad z > l(z) ,$$
$$l(z) = f_w(z) , \qquad z \le \frac{4<m(z)>}{S_w(z)} . \tag{54}$$

The set of governing equations consists of highly nonlinear second-order ordinary and partial parabolic differential equations. Solutions for the equation set were obtained mainly using finite difference methods described in papers by Travkin (1981, 1984, 1985). The finite difference schemes have incorporated a special local augmentation to deal with both numerical dissipation and diffusion. Solutions of the set of equations are sought using an uneven gridding and discontinuous coefficients. The last implementation is desirable because one can treat the highly variable or random coefficients in the equations. Most of the calculations were performed using 201 nodes across the layer. Comparisons of 200 node calculations with 401 and 501 nodes show local difference to be less then 1-3%. The Nusselt number at the wall is defined

$$Nu_w = \left(- \left(\frac{\partial T}{\partial z} \right) \frac{h}{(T_w - T_{fm})} \right) \Big|_{z = +0} , \tag{55}$$

and inside of the porous medium

$$Nu_z = \tilde{\alpha}_T (z) h / K_T(z) , \quad 0 \le z \le h . \tag{56}$$

6 Results and Discussion

The thermophysical properties an air - steel porous media were chosen for numerical simulation with temperature boundary similarity number Q_{w2} scaled as 1. The regular and random porous structure of spherical packing were introduced by taking the coefficient correlations, while the mathematical statements in Eqs. (10), (11), (18) - (21) were developed with the respect of only regular particle locations. At the present time, there is little reliable data showing the difference between regular and random and regular (periodic) structure of the porous medium granules. For most of the calculated results the differences between the experimental pressure drop for spherical randomly located particles and the present work using experimental drag resistance correlations were less than 1-3% due to the overall bulk character of the experimental correlations. Fig. 2 presents comparisons of some profiles for tube and nonstructured porous medium morphology models. It can be seen that it is inappropriate to use canonical morphology models as simple as regular straight tubes to represent a highly porous media morphology.

Fig. 3 shows the profiles of drag resistance coefficient for different porous medium morphology. The difference in an admitted morphology structure resulted in four order change of magnitude of the resistance similarity number c_{dm} and consequently resulted in big differences in the temperature and Nusselt numbers. The Nusselt numbers as well as temperatures of the solid and fluid phases and other characteristics along the channel are seen to depend strictly on the drag resistance, c_{dm} , and morphology, A_4 , functions, (see Figs. 4 and 5). These figures present the influence of varying the porosity value for periodic spherical packing. Generalization of experimental measurements of the effective thermal conductivity of fluids in a porous medium are given by Aarov et al. (1979)

$$k_{f,e} = K_{ex}\,\rho_f\, c_{pf}\,\tilde{u}\; d_{ch}, \quad K_{ex} = 0.07 - 0.11 . \tag{57}$$

The experiments by Kharitonov et al. (1987) showed that $K_{ex}= 0.075\pm0.009$ for water in the range $Re_{ch} = 11 - 1500$. The coefficient of effective thermal conductivity in a fluid, calculated as the coefficient of turbulent eddy thermal conductivity in this work,

$$k_{f,e} = m_0\,(\tilde{K}_T + k_f)\,, \tag{58}$$

yielded values within ten to twenty percent without modification of any important constants C_1 nor the value of the mixing length. The initial value of the first was taken from boundary layer theory, whereas the second is the simple "effective" pore diameter in a regular spherical porous medium. There are three theoretical investigations, that are close to the present work mathematical statement for low velocity regime. All are analytical solutions. In the paper of Plakseev and Kharitonov (1988), a 2-temperature model for a porous layer is solved for constant coefficients with an assigned heat flux on one wall, a restricted II-d kind boundary conditions on the other wall, and a constant value of velocity along the layer. In the work of Vafai and Kim (1989) (see also Vafai and Thiyagaraja, 1987) primary interest was paid to an approximate analytical solution of the momentum equation with constant coefficients. The analytical solution for the combined momentum equation of Vafai and Kim (1989) was compared to the mean values of velocity and gave the difference less than 3% for slow regimes.

Solution of the momentum part of the model statement, Eqs. (48) - (50), allows one to obtain an effective thermal conductivity of the fluid. It was incorporated into the weighted value of the effective 1-temperature thermal conductivity of a homogenized medium. In spite of large, and sometimes much larger effective thermal coefficients of conductivity in 1-temperature model, the values of the homogeneous temperature in a 1-temperature model were much smaller for the air-steel porous medium physical characteristics (Fig. 6). One should not be surprised by this result, because in models of the air-steel pair, analyzed in this work, the parameter

$$\alpha_s = (\,K_{ST}\,(1\, -\, m_0)\;\tilde{\alpha}_{T,m}\, S_w\,)^{1/2}\,, \tag{59}$$

is the mean coefficient of heat transfer and the maximum capacity of the solid phase to extract energy to the fluid phase (Plakseev and Kharitonov, 1988), is much larger than the coefficient of heat transfer from the wall α_w. The latter was calculated independently from the numerical results of this work and from the correlations of Kokorev et al. (1987) and Aarov et al. (1979).The magnitude of α_s calculated in the present work for air-steel is $\alpha_s = 0(10^n)$, n=1-2 and it means that almost all heat transfer from the wall is transferred by the solid phase. Also, according to Plakseev and Kharitonov (1988) the ratio $r_k = K_{ST}(1-m_0)/k_{f,e} > 10$ denotes that most of heat is transmitted to the fluid by means of the heat exchange inside of the porous medium not at its boundaries. Similar result was obtained by Vafai and Sozen (1990) for heat transfer with constant wall temperature in a packed bed formed by the pair Freon-12 - steel particles.

The sensitivity of the present model to the morphological characteristics of the porous medium is quite evident while providing the basis for the next numerical experiments. Calculations using the local porosity and specific surface distributions for a regular spherical cubic medium were compared to similar calculations where the only difference is calculation of the overall drag resistance coefficient. Calculations using a c_d model that depends on the local porosity distribution function were compared to the results of experimentally determined c_d values for a single spherical particle. In the latter model, c_d depends on the local particle Reynolds number over a wide range of Reynolds number dependent correlations. When used with the analytical local porosity and specific surface expressions, the model predicted oscillations in the main quantities, such as the drag coefficient and velocity distribution. The single spherical particle drag resistance model displayed both drag coefficient and velocity profile characteristics of beds with constant bulk morphology functions. It appears that while the first model uses the local morphology characteristic variation dependence, the second consists of dependence of the drag resistance only on the bead diameter. More results on variation dependencies of porous medium morphology are given in an article by Gratton et al. (1993).

The present results were compared with the numerical results of Poulikakos and Renken (1987) for flow and heat transfer with constant coefficients inside of a porous layer. The differences shown in Fig. 7 (where the variant of that by Poulikakos and Renken, 1987, is demonstrated with dimensionless parameters
$B= -(h^3/(D<^2))(dp/dx)=10^5$, dimensionless sphere diameter $d_*=d_p/h=0.1$, $d_p=0.005$m, $m_0=0.37$) are due to the differences of substantiation of the initial equations and particularly because of a rather contrary interpretation of resistance terms, as was shown in section 5 of this work. The mean characteristics of the comparison are relatively close, with differences being better observed in the boundary layer.

7 Summary

A non-linear model for two-temperature heat and momentum turbulent transport is presented. All the coefficient models used and discussed in this work are strictly for assumed (or admitted) porous medium morphological models based on well described geometry. This approach shows that to model the morphology of a porous media, the coefficients in the equations, as well as the equation's forms itself must be consistent. The integral terms in the equations can be dropped or transformed depending on the porous medium structure, flow and heat transfer regimes. Developed closure approach allows one to obtain exact analytical closure dependencies for simple porous medium morphologies. Numerical analysis made with the help of parameters suggested by Plakseev and Kharitonov (1988) for heat exchange between wall - fluid, solid - fluid phases shows the impact of heat exchange correlations for $''_s$, $''_w$, r_k and reveals the combinations of similarity numbers that depict almost total heat removal from the wall by the solid phase for air-steel porous medium while an intense flow regime occurs.

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References

Aarov, M. A., Todes, O. M., and Narinsky, D. A., 1979, *Apparatuses with Stationary Granular bed*, Chemistry, Leningrad.

David, E., Lauriat, G., and Cheng, P., 1991, "A Numerical Solution of Variable Porosity Effects on Natural Convection in a Packed-Sphere Cavity," *ASME Journal of Heat Transfer*, Vol. 113, No. 2, pp. 391-399.

Dullien, F. A. L., 1992, *Porous Media. Fluid Transport and Pore Structure,* Academic Press, New York.

Du Plessis, J. P., and Masliyah, J. H., 1988, "Mathematical Modelling of Flow Through Consolidated Isotropic Porous Media," *Transport in Porous Media*, Vol. 3, No. 2, pp. 145-161.

Ergun, S., 1952, "Fluid Flow Through Packed Columns," *Chemical Engineering Progress*, Vol. 48, pp. 89-94.

Fand, R. M., and Thinakaran, R., 1990, "The Influence of the Wall on Flow Through Pipes Packed with Spheres," *Journal of Fluids Engineering*, Vol. 112, No. 1, pp. 84-88.

Georgiadis, J. G., and Catton, I., 1986, "Prandtl Number Effect on Benard Convection in Porous Media," *ASME Journal of Heat Transfer*, Vol. 108, No. 2, pp. 284-290.

Georgiadis, J. G., and Catton, I., 1987a, "Stochastic Modeling of Unidirectional Fluid Transport in Uniform and Random Packed Beds," *Physics of Fluids*, Vol. 30, No. 4, pp. 1017-1022.

Georgiadis, J. G., and Catton, I., 1987b, "Transport in Pipes Filled with Uniform, Non-Uniform and Random Porous Media," *Proceedings of the Second ASME/JSME Thermal Engineering Joint Conference*, Vol. 2, pp. 395-400.

Georgiadis, J., Behringer, R., Shattuck, M., and Johnson, G. A., 1991, "Interstitial Velocity and Temperature Fields in Fully-Saturated Porous Media," *Proceedings of the Ninth Symposium on Energy Engineering Sciences, Fluid and Dynamical Systems,* Argonne National Laboratory, Argonne, pp. 101-109.

Gratton, L., Travkin, V. S., and Catton, I., 1993, "Transport coefficient dependence upon solid phase morphology for single phase convective transport in porous media," *Proceedings of the 29th National Heat Transfer Conference,* HTD-240, ASME, Atlanta, pp. 11-21.

Hayes, R. E., 1990, "Simulation of Mixed Convection Heat Transfer at the Wall of a Packed Bed," *Numerical Heat Transfer: Part A*, Vol. 17, No. 2, pp. 217-230.

Heat Exchanger Design Handbook, 1983, (Spalding, B.D., Taborek, J., Armstrong, R.C. and et al., contribs.), N.Y., Hemisphere Publishing Corporation, Vol.1,2.

Hunt, M. L., and Tien, C. L., 1988a, "Non-Darcian Convection in Cylindrical Packed Beds," *ASME Journal of Heat Transfer*, Vol. 110, No. 2, pp. 378-384.

Hunt, M.L., and Tien, C.L., 1988b, "Effects of Thermal Dispersion on Forced Convection in Fibrous Media," *International*

Journal of Heat and Mass Transfer, Vol. 31, No. 2, pp. 301- 309.
Immich, H., 1980, "Impulsive Motion of a Suspension: Effect of Anti-Symmetric Stresses and Particle Rotation," *International Journal of Multiphase Flow*, Vol. 6, pp. 441-471.
Kader, B. A., 1977, "Hydraulic Resistance of Surfaces Covered with Two-Dimensional Roughness with Large Reynolds Numbers," *Theoretical Foundations of Chemical Engineering*, Vol. 11, No. 3, pp. 324-333.
Kaguei, L, Tanisho, L., and Wakao, N., 1983, "Particle-to-Fluid Heat Transfer Coefficients at Low Flow Rates in Packed Beds -- Approaching a Limited Value or Unlimited Decline?," *Third Pacific Chemical Engineering Congress*, Seoul, Korea, May 8-11, proceedings Vol. 2, pp. 7-12.
Kaviany, M., 1985, "Laminar Flow Through a Porous Channel Bounded by Isothermal Parallel Plates", *Intern. J. Heat Mass Transfer*, Vol. 28, No. 4, pp.851-858.
Kharitonov, V. V., Plakseev, A. A., Fedoseev, V. N., and Voskoboinikov, V. V.,., 1987, "Fluid Flow Mixing Influence on the Heat Transfer in the Porous Insert Channels", *Teplopfizika Visokikh Temperatur*, Vol. 25, No. 5, pp. 954-961.
Kheifets, L. I., and Neimark, A. V., 1982, *Multiphase Processes in Porous Media*, Nedra, Moscow.
Kladias, N., and Prasad, V., 1990, "Flow Transitions in Bouyancy Induced Non-Darcy Convection in a Porous Medium Heated from Below," *ASME Journal of Heat Transfer*, Vol. 112, No. 3, pp. 675-684.
Klenov, O.P. and Matros, Yu.Sh., 1990, "Effect of Loading Conditions on the Porosity and Hydraulic Resistance of a Stationary Granular Bed", *Theoretical Foundations of Chemical Engineering*, Vol. 24, No. 2, pp. 206-210.
Kokorev, V. I., Subbotin, V. I., Fedoseev, V. N., Kharitonov, V. V., and Voskoboinikov, V. V., 1987, "On Interconnection of Hydraulic Resistance and Heat Transfer in Porous Media," *High Temperature*, Vol. 25, No. 1, pp. 82-87.
Menzhulin, G. V., 1970, "On Methodic of Meteorology Regime Calculation in Plant Community," *Meteorology and Hydrology*, No. 2, pp. 92-99.
Monin, A.S., and Yaglom, A.M., 1975, *Statistical Fluid Mechanics*, Vol. 1,2, MIT Press, Cambridge, Massachusetts.
Nield, D.A., 1991, "The limitations of the Brinkman-Forchheimer equation in modeling flow in a saturated porous medium and at an interface", *International Journal of Heat and Fluid Flow*, Vol. 12, No. 3, pp. 269-272.
Plakseev, A. A., and Kharitonov, V. V., 1988, "Heat Transfer in Channels with Porous Inserts during Forced Fluid Flow," *Journal of Engineering Physics*, Vol. 56, No. 1, pp. 26-33.
Poulikakos, D., and Renken, K., 1987, "Forced Convection in a Channel Filled With Porous Medium, Including the Effects of Flow Inertia, Variable Porosity, and Brinkman Friction", *ASME Journal of Heat Transfer*, Vol. 109, No. 4, pp. 880-888.
Scherban, A. N., Primak, A.V., and Travkin, V. S., 1986, "Mathematical Models of Flow and Mass Transfer in Urban Roughness Layer," *Problemy Kontrolya i Zaschita Atmosfery ot Zagryazneniya*, No. 12, pp. 3-10.
Shvidler, M. I., 1986, "Conditional Averaging of Unsteady Percolation Fields in Random Composite Porous Media," *Fluid Dynamics*, Vol. 21, No. 5, pp. 735-740.
Shvidler, M. I., 1989, "Average Description of Immiscible Fluid Transport in Porous Media with Small-Scale Inhomogeneity.," *Fluid Dynamics*, Vol. 24, No. 6, pp. 902-909.
Taylor, R. P., Coleman, H. W., and Hodge, B. K., 1985, "Prediction of Turbulent Rough-Wall Skin Friction Using a Discrete Element Approach," *Journal of Fluids Engineering*, Vol. 107, No. 2, pp. 251-257.
Travkin, V. S., 1981, "Steady Free Convection in the Cavity With Variable Wall Temperature", *Heat Exchange in Mono- and Biphase Media*, Naukova Dumka, Kiev, pp.15-19.
Travkin, V.S., 1984, "Numerical Simulation of Heat and Mass Transfer in a Continuous-Casting Mold", *High Temperature*, Vol.22, No.2, pp.251-258.
Travkin, V.S., 1985, "Numerical Integration of Free Convection Compressible Gas Equations in Enclosed Cylindrical Cavity on Conjugated Statement", *Numerical Methods of Continuous Medium Mechanics*, Vol.16, No.3, pp.128-152.
Travkin, V., and Catton, I., 1992a ,"Turbulent Forced Transport Governing Equations in High Permeability Medium," submitted to the *Physics of fluids*, Sec A.
Travkin, V. and Catton, I., 1992b, "Models of Turbulent Thermal Diffusivity and Transfer Coefficients for a Regular Packed Bed of Spheres", *Proceedings 28th National Heat Transfer Conf.*, ASME, HTD-Vol. 193, pp.15-23.
Travkin, V. and Catton, I., 1992c, "The Numerics of Turbulent Processes in High Porosity Nonuniform Porous Media", *Proceedings of the SIAM 40th Anniversary Meeting*, Los Angeles, pp.A36.
Travkin, V., and Catton, I., 1994a, "Simulations of Turbulent Transport in Some Kinds of Highly Variable Non-isotropic and Bi-porous Media", (will be submitted to the "*Journal of Heat Transfer*").
Travkin, V., Catton, I., and Gratton, L., 1994b, "The convergency of two-temperature transfer model to one temperature in a porous medium with determined morphology and pertinent external boundary conditions", (will be submitted to the "*Journal of Heat Transfer*").
Vafai, K., and Tien, C.L., 1981, "Boundary and Inertia Effects on Flow and Heat Transfer in Porous Media," *International Journal of Heat and Mass Transfer*, Vol. 24, pp.195-203.
Vafai, K., and Thiyagaraja, R., 1987, "Analysis of Flow and Heat Transfer at the Interface Region of a Porous Medium", *Int. J. Heat Mass Transfer,* Vol. 30, No. 7, pp.1391-1405.
Vafai, K., and Kim, S. J., 1989, "Forced Convection in a Channel Filled with a Porous Medium: An Exact Solution," *ASME Journal of Heat Transfer*, Vol. 111, No. 4, pp. 1103-1106.
Vafai, K., and Sozen, M., 1990, "Analysis of Energy and Momentum Transport for Fluid Flow Through a Porous Bed", *J. Heat Transfer,* Vol. 112, pp.690-699.

Ward, J. C., 1964, "Turbulent Flow in Porous Media," *Journal of the Hydraulics Division, Proceedings of the ASCE*, Vol. 90, HY5, pp. 1-12.

Watanabe, H., 1989, "Drag Coefficient and Voidage Function on Fluid Flow through Granular Packed Beds," *International Journal of Engineering Fluid Mechanics*, Vol. 2, No. 1, pp. 93-108.

Whitaker, S., 1967, "Diffusion and Dispersion in Porous Media," *AIChE Journal*, Vol.13, No. 3, pp. 420-427.

Appendix A

Turbulent transport equation set for the first level of hierarchy - in a pore was taken as next

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 , \qquad \text{(A1)}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = - \frac{1}{\rho_f} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \bar{u}_i}{\partial x_j} - \overline{u_i' u_j'} \right) + S_u , \qquad \text{(A2)}$$

$$\frac{\partial \bar{T}}{\partial t} + \bar{U}_i \frac{\partial \bar{T}}{\partial x_i} = \frac{\partial}{\partial x_i} \left(\lambda (\vec{x} , \bar{T}, \bar{U}) \frac{\partial \bar{T}}{\partial x_i} - \overline{u_i' T'} \right) + S_T , \qquad \text{(A3)}$$

where T represent any scalar field (temperature) that might be transported into either of the porous medium phases, and last terms on the right hand side of (A2) and (A3) are source terms.

This nonlinear set of equations has been averaged using developed nonlinear averaging technique (Scherban et al., 1986; Travkin and Catton, 1992a). Formalism of equation averaging in heterogeneous media differ from of the known ones in the account of different scale fluctuations, in the second averaging models and also in the differential operators averaging.

Turbulent quantities in the averaged equations are decomposed as follows

$$\begin{gathered} U = \bar{U} + u' = \bar{U}_k + \tilde{U}_r + u_k' + u_r' , \\ \bar{U} = \tilde{U} + \hat{u} \quad , \quad \hat{u} = u_r' , \end{gathered} \qquad \text{(A4)}$$

where the index "k" means for the turbulent components independent of nonhomogeneities of dimensions and properties of the multitude of porous media channels (pores), and "r" denotes the contributions due to the porous medium inhomogeneity. Turbulent kinetic energy equation (10) and averaged equation for mean eddy diffusivity (11) complement and close mathematical statement for 2D (b - l) turbulent transport model in highly porous medium with regular morphology.