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Exact Closure Procedures of Hierarchical VAT Capillary Thermo-Convective Problem for Turbulent and Laminar Regimes

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ABSTRACT

Volume Averaging Theory (VAT), an effective and rigorous approach for study of transport (laminar and turbulent) phenomena, is used to model flow and heat transfer in porous media. The modeling is based on a simple pore level network. The primary difficulties in applying VAT to straight capillary networks, the many unknown integral and differential terms that are needed for closure, are overcome. VAT based modeling of pore level transport in straight capillaries results in two sets of scale governing equations. One scale is the upper scale VAT equations which describe ensemble properties for flow and heat transfer in porous media. The other scale is the lower scale laminar and turbulent transport equations that represent flow and heat transport in each straight pore capillary. It is how the unknown VAT terms in the upper scale equations can be estimated using the relationships between upper scale properties and lower scale properties. Exact closures and mathematical procedures are developed for the turbulent regime, extending the previous laminar regime work. Numerical results for turbulent and laminar transport in straight capillary porous media are shown in this paper.

Nomenclature

c_p	-	specific heat $[J/(kg \cdot K)]$
d_h	-	hydraulic dynameter, $[m]$
ds	-	interphase differential area in porous medium $[m^2]$
∂S_w	-	internal surface in the REV $[m^2]$
f	-	friction factor
$\widetilde{f}$	-	$= \frac{1}{\Delta\Omega_f}\int_{\Delta\Omega_f} f d\omega$, averaged over $\Delta\Omega_f$ value f
$< f >_f$	-	$= \frac{1}{\Delta\Omega}\int_{\Delta\Omega_f} f\left(\overrightarrow{X}, t\right) d\omega$, value f, averaged over $\Delta\Omega_f$ in a REV $\Delta\Omega$
$\widehat{f}$	-	$= f - \widetilde{f}$, value f local morpho-fluctuation in a $\Delta\Omega_f$
k	-	thermal conductivity $[W/(mK)]$
$\widetilde{\nu}_T$	-	averaged turbulent eddy viscosity $[m^2/s]$

k_{sT} - effective thermal conductivity of solid phase $[W/(mK)]$
k_T - turbulent eddy thermal conductivity $[W/(mK)]$
m - porosity [-]
$\langle m \rangle$ - averaged porosity [-]
N - Total number of pores
P_r - Prandtl number [−]
p - pressure $[Pa]$
q - heat flux $[W/m^2]$
R_i - i^{th} pore radius $[m]$
Re_h - Reynolds number of pore hydraulic diameter [-]
S_{cr} - cross section flow area $[m^2]$
S_i - i^{th} pore cross section flow area $[m^2]$
S_w - specific surface of a porous medium $\partial S_w/\Delta\Omega$ $[1/m]$
T - temperature $[K]$
u, w - velocity in x,z-direction $[m/s]$

Subscripts

f - fluid phase
i - component of turbulent vector variable, i^{th} pore
L - laminar, lower scale
s - solid phase
T - turbulent
w - wall
win - inner wall
$wout$ - outer wall

Superscripts

$\sim$ - intrinsic value in fluid phase averaged over the $\Delta\Omega_f$
$-$ - mean turbulent quantity
u - upper scale

Greek letters

$\Delta\Omega$ - representative elementary volume (REV) $[m^3]$
$\Delta\Omega_f$ - pore volume in a REV $[m^3]$
$\Delta\Omega_s$ - solid phase volume in a REV $[m^3]$
ν - kinematic viscosity $[m^2/s]$
ϱ - density $[kg/m^3]$
τ - turbulent friction stress tensor $[N/m^2] = -\varrho\overline{u'w'}$
τ_w - wall shear stress $[N/m^2]$

Introduction

Volume averaging is a widely used technique in which a macroscopic momentum equation is derived from Navier-Stokes turbulence equations by averaging over a representative elementary volume (REV). During the averaging process, hydrodynamic information from the pore scale is retained in the form of unknown surface expressions and fluctuations that can be determined experimentally or derived exactly for simple pore structures.

A popular pore level model used to study the porous media is the network model. Network models use a series of interconnected nodes and bonds with distributed sizes. In a network model the pore space is represented as a graph of connected sites. A common interpretation of this graph is that the sites correspond to pore bodies, and the bonds correspond to pore throats connecting the pore bodies. In principle, a network model can replicate both the geometry and topology of the pore space, so that flow through the network is equivalent to flow through the actual porous medium.

The parameters that dictate a network's geometry are its spatial dimension (i.e., 2-D or 3-D), grid pattern (which maybe regular or irregular), bond-size distribution, and coordination number (the value and whether it is constant). The development of a network model for flow in porous media began with Fatt (1956) who used an equivalent resistor network to calculate properties like capillary pressure, rela-

tive permeability, and resistance. Since then, numerous studies have been carried out with increasingly sophisticated rules to describe capillary equilibrium and simultaneous flow of fluids. of these studies, those by Goode and Ramakrishnan (1993), Thompson and Fogler (1997), Rieckmann and Keil (1997) and Thauvin and Mohanty (1998) are of particular interest.

To develop valid VAT equations, a straight capillary network is our starting point. It is a very simple network. There is any junction in this network. The advantage of this network is that flow and heat transfer in each pore can easily be simulated. Using the results in each pore, the VAT equations can easily be verified and exact closure of the VAT equations can be developed. In this paper, the capillary morphology, shown in Fig. 1, is used as a morphology model for volume averaged network model development. Travkin and Catton (1999) obtained a two scale solution for the volume averaging theory (VAT) model of momentum transport in a simple case of straight capillaries. For heat transfer in a network morphology, the temperature field is not homogeneous. A two scale solution for energy transport is addressed in this work. This work is an extension of Travkin and Catton (1999) and Hu et al. (2001).

The problem of lower scale heat transport in a bundle of straight capillaries is a conjugate problem. The analytical methods of solution of conjugated problems presented by Luikov et al. (1970) and Luikov (1974) are used in this paper for solving the lower scale conjugated convective heat transfer problems. Previous studies have shown that VAT models are effective for the study of upper scale heat transport in straight capillary morphology, see Travkin and Catton (1998), Zanotti and Carbonell (1984) and Yuan et al. (1991). The approach used in this paper, however, to formulate closure and

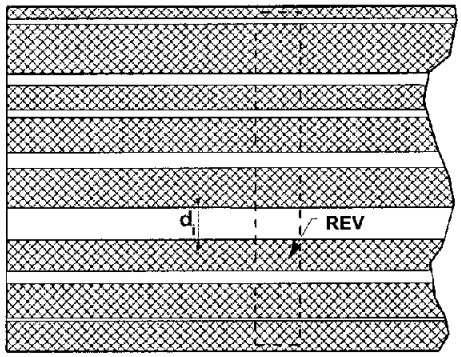

Figure 1: Capillary morphology model of porous medium: a bundle of parallel pores embeded in solid

to find a solution of the current VAT problem is essentially different from the previous work.

Momentum Transport

The one dimensional straight pore morphology (SPM) shown in Fig. 1 (Travkin and Catton, 1999) is chosen to show how model equations and consistent closure models based on VAT are developed for transport of momentum in a porous media using a network model. For SPM, theoretical solutions for bulk velocity and dispersion coefficients can be obtained.

Upper Scale Momentum Equation

The VAT based 1-D momentum equation for flow in porous medium shown in Fig. 1 is

$$\frac{\partial}{\partial x}\left(\langle m\rangle\left(\widetilde{\nu}_T+\nu\right)\frac{\partial\widetilde{\overline{u}}}{\partial x}\right) \quad (1)$$
$$+\frac{\partial}{\partial x}\left(\left\langle\widetilde{\nu}_T\frac{\partial\widehat{\overline{u}}}{\partial x}\right\rangle_f\right)+\frac{\partial}{\partial x}\left(\left\langle-\widehat{\overline{u}}\,\widehat{\overline{u}}\right\rangle_f\right)$$
$$= \langle m\rangle\widetilde{\overline{u}}\frac{\partial\widetilde{\overline{u}}}{\partial x_j}+\frac{1}{\varrho_f\Delta\Omega}\int_{\partial S_w}\overline{p}\,\vec{ds}$$
$$+\frac{1}{\varrho_f}\frac{\partial}{\partial x}\left(\langle m\rangle\widetilde{\overline{p}}\right)-\frac{1}{\Delta\Omega}\int_{\partial S_w}\left(\nu_T+\nu\right)\frac{\partial\overline{U}_i}{\partial x_i}\cdot\vec{ds}$$

where ν_T is the turbulent eddy viscosity. It is necessary to take the surface stress as three dimensional in the last term of the equation. Because the interface surface ∂S_w might locally be 3D and closure demands that the local lower level features of ∂S_w be treated rigorously. The closure scheme for this equation is explained in Travkin and Catton (1999).

Lower Scale Momentum Transport in Pores

Based on the general scheme developed by Travkin and Catton (1995, 1998), the instant velocity for the turbulent regime in straight, smooth, nonintersecting pores, is represented by

$$\begin{aligned} u &= \overline{u}(x,r)+u'(x,r,t) \quad (2)\\ &= \overline{u}_k(x,r)+\widetilde{u}_r(x,r)+u_r'(x,r)+u_k'(x,r,t)\end{aligned}$$

where

$$\begin{aligned}\overline{u}(x,r) &= \frac{1}{t_T}\int_{t_T}u\,dt \quad (3)\\ &= \widetilde{\overline{u}}(x,r)+u_r'(x,r)\\ &= \overline{u}_k(x,r)+\widetilde{u}_r(x,r)+u_r'(x,r)\\ u_r'(x,r) &= \widehat{\overline{u}}(x,r)\end{aligned}$$

and also

$$\widetilde{\overline{u}}(x,r)=\sum_{i=1}^{N}\overline{\overline{u}}_{ki}S_i/S_{cr} \quad (4)$$

where k signifies turbulence induced components that are independent of inhomogeneities in spatial upper scale morphology features and properties resulting from the multitude of porous medium channels (pores), and r denotes the fluctuation contribution due to the porous medium inhomogeneity. The relationship between the lower scale velocity and the upper scale velocity, and the relationship between average velocity and fluctuation velocity are shown in Fig. 2 in Travkin and Catton (1999). From that figure, it can be derived that the mean velocity in each pore i is

$$\begin{aligned}\overline{\overline{u}}_{ki}(r) &= \frac{2\pi}{\pi R_i^2}\int_0^{R_i}\overline{u}_i r\,dr \quad (5)\\ &= \frac{2\pi}{\pi R_i^2}\int_0^{R_i}\left(\widehat{\overline{u}}_i+\widetilde{\overline{u}}\right)r\,dr\\ &= \widetilde{\overline{u}}+a_i\end{aligned}$$

$$\begin{aligned}\widehat{\overline{u}}_i(r) &= 2\overline{\overline{u}}_{ki}(r)\left[1-\left(\frac{r}{R_i}\right)^2\right]-\widetilde{\overline{u}} \quad (6)\\ &= \widetilde{\overline{u}}\left[1-2\left(\frac{r}{R_i}\right)^2\right]+2a_i\left[1-\left(\frac{r}{R_i}\right)^2\right]\end{aligned}$$

and

$$\overline{u}_i(r)=2\overline{\overline{u}}_{ki}(r)\left[1-\left(\frac{r}{R_i}\right)^2\right] \quad (7)$$

For a two pore structure, when the average in the REV is defined over the averaging volume $\Delta\Omega_f=\Delta\Omega_{f1}+\Delta\Omega_{f2}$, the averaged velocity in the REV is given by

$$\widetilde{\overline{u}}=\frac{1}{\Delta\Omega_f}\left(\int_{\Delta\Omega_{f1}}\overline{u}_1 d\omega+\int_{\Delta\Omega_{f2}}\overline{u}_2 d\omega\right) \quad (8)$$

For laminar flow in a single pore it can easily be shown that

$$\begin{aligned}\overline{\overline{u}}_{ki} &= \frac{1}{\pi R_i^2}\int_0^{R_i} u_i(r)\,2\pi r dr \qquad (9)\\ &= -\frac{1}{8\mu}\left(\frac{dp}{dx}\right)R_i^2\end{aligned}$$

For turbulent flow in a single pore i with constant properties, the momentum conservation equation is

$$\frac{1}{r}\frac{d}{dr}\left[r\left(\nu+\nu_T\right)\frac{d\overline{u}_i(r)}{dr}\right]=\frac{1}{\varrho}\left(\frac{dp}{dx}\right) \qquad (10)$$

The solution of Eq. (10) is initiated by regarding the pressure gradient as known and constant. The rest of the solution procedure follows as

$$\tau_w=\left(-\frac{dp}{dx}\right)\frac{\pi R_i^2}{2\pi R_i}=\frac{R_i}{2}\left(-\frac{dp}{dx}\right), \qquad (11)$$

$$Re_h=\frac{\overline{\overline{u}}_{ki}(2R_i)}{\nu} \qquad (12)$$

$$f_f=\frac{2\tau_w}{\varrho_f\overline{\overline{u}}_{ki}^2}=\frac{2R_i}{2\varrho_f\overline{\overline{u}}_{ki}^2}\left(-\frac{dp}{dx}\right)=\frac{0.079}{\mathrm{Re}_h^{0.25}} \qquad (13)$$

$$u^*=\left(\frac{\tau_w}{\varrho_f}\right)^{\frac{1}{2}}=\overline{\overline{u}}_{ki}\left(\frac{f_f}{2}\right)^{\frac{1}{2}}, \qquad (14)$$

So

$$\begin{aligned}\overline{\overline{u}}_{ki} &= \left[\frac{2R_i}{2\varrho_f\, f_f}\left(-\frac{dp}{dx}\right)\right]^{\frac{1}{2}} \qquad (15)\\ &= \left[\frac{2R_i}{2\varrho_f\,\frac{0.079}{\mathrm{Re}_h^{0.25}}}\left(-\frac{dp}{dx}\right)\right]^{\frac{1}{2}}\end{aligned}$$

Solving Eq. (15) yields $\overline{\overline{u}}_{ki}$.

If the inner variable y^+, defined as

$$y^+=\frac{yu^*}{\nu}=\left(\frac{(R_i-r)\,u^*}{\nu}\right), \qquad (16)$$

is introduced, then the velocity in each pore is of the form (Schlichting, 1979)

$$\begin{aligned}&\overline{u}_i(r) \qquad (17)\\ &= u^*5.75lg\left[\left(1+0.4y^+\right)\frac{1.5\,(1+R_i)}{1+2R_i^2}\right]\\ &\quad +7.8\left[1-exp\left(-\frac{y^+}{11}\right)-\frac{y^+}{11}exp\left(-\frac{y^+}{3}\right)\right],\end{aligned}$$

where R_i is the radius of i^{th} pore. The turbulent viscosity is given by

$$\begin{aligned}\nu_{Ti}\left(y^+\right) &= 0.07\nu\left(y^+-11th\left(\frac{y^+}{11}\right)\right)\\ &\quad (1-R_i)\left(1+2R_i^2\right) \qquad (18)\end{aligned}$$

Heat Transfer Analysis for Network Model

For heat transfer in the medium shown in Fig. 1, the temperature field is not homogeneous. The two temperature model developed in Travkin and Catton (1995) should be used as the upper scale governing equation.

Upper Scale Governing Equations

General statements for energy transport in a porous medium require two-temperature treatments. Travkin et al. (1993) showed that the proper form for the turbulent heat transfer equation in the fluid phase using K-theory one-equation closure with primarily 1-D convective heat transfer is

$$
\begin{aligned}
& c_{pf}\varrho_f \langle m\rangle \widetilde{\overline{U}}\frac{\partial \widetilde{\overline{T}}_f}{\partial x} \qquad (19)\\
= \; & \frac{\partial}{\partial x}\left[\langle m\rangle \left(\widetilde{k}_{_T}+k_f\right)\frac{\partial \widetilde{\overline{T}}_f}{\partial x}\right]\\
& +\frac{\partial}{\partial x}\left(\left\langle \widehat{k}_{_T}\frac{\partial \widehat{\overline{T}}_f}{\partial x}\right\rangle_f\right)\\
& +c_{pf}\varrho_f\frac{\partial}{\partial x}\left[\langle m\rangle\left\{-\widehat{\overline{T}}_f\,\widehat{\overline{u}}\right\}_f\right]\\
& +\frac{\partial}{\partial x}\left[\frac{\left(\widetilde{k}_{_T}+k_f\right)}{\Delta\Omega}\int_{\partial S_w}\widehat{\overline{T}}_f\,\vec{ds}\right]\\
& +\frac{1}{\Delta\Omega}\int_{\partial S_w}\left(k_{_T}+k_f\right)\frac{\partial \overline{T}_f}{\partial x_i}\cdot\vec{ds},
\end{aligned}
$$

while in the neighboring solid phase, the corresponding equation is

$$
\begin{aligned}
& \frac{\partial}{\partial x}\left[(1-\langle m\rangle)\left\{k_{sT}\right\}_s\frac{\partial\left\{T_s\right\}_s}{\partial x}\right] \qquad (20)\\
& +\frac{\partial}{\partial x}\left(\left\langle \widehat{k}_{sT}\frac{\partial \widehat{T}_s}{\partial x}\right\rangle_s\right)\\
& +\frac{\partial}{\partial x}\left[\frac{\left\{k_{sT}\right\}_s}{\Delta\Omega}\int_{\partial S_w}\widehat{T}_s\,\vec{ds}_{_1}\right]\\
& +\frac{1}{\Delta\Omega}\int_{\partial S_w}k_{sT}\frac{\partial T_s}{\partial x_i}\cdot\vec{ds}_{_1}\\
= \; & 0.
\end{aligned}
$$

For laminar flow, the two-temperature energy equation can be simplified to

$$
\begin{aligned}
& c_{pf}\varrho_f \langle m\rangle \widetilde{U}\frac{\partial \widetilde{T}_f}{\partial x} \qquad (21)\\
= \; & \frac{\partial}{\partial x}\left[\langle m\rangle k_f\frac{\partial \widetilde{T}_f}{\partial x}\right]\\
& +c_{pf}\varrho_f\frac{\partial}{\partial x}\left[\langle m\rangle\left\{-\widehat{T}_f\,\widehat{u}\right\}_f\right]\\
& +\frac{\partial}{\partial x}\left[\frac{k_f}{\Delta\Omega}\int_{\partial S_w}\widehat{T}_f\,\vec{ds}\right]\\
& +\frac{1}{\Delta\Omega}\int_{\partial S_w}k_f\frac{\partial T_f}{\partial x_i}\cdot\vec{ds},
\end{aligned}
$$

and in the neighboring solid phase, the corresponding equation is

$$
\begin{aligned}
& \frac{\partial}{\partial x}\left[(1-\langle m\rangle)\,k_s\frac{\partial T_s}{\partial x}\right] \qquad (22)\\
= \; & -\frac{\partial}{\partial x}\left[\frac{k_s}{\Delta\Omega}\int_{\partial S_w}\widehat{T}_s\,\vec{ds}_{_1}\right]\\
& -\frac{1}{\Delta\Omega}\int_{\partial S_w}k_s\frac{\partial T_s}{\partial x_i}\cdot\vec{ds}_{_1}
\end{aligned}
$$

where

$$\vec{ds}_1=-\vec{ds}$$

Lower Scale Governing Equations

At the lower scale, the flow and heat transfer in each pore is homogeneous allowing the governing equations to be simplified. One feature is different, spatially it is 2D. For convenience, the equations are written in cylindrical coordinates instead of Cartesian coordinates.

Governing Equation in the Solid Phase In the solid phase, the two dimensional steady state conduction equation can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_{s_i}}{\partial r}\right)+\frac{\partial}{\partial x}\left(\frac{\partial T_{s_i}}{\partial x}\right)=0 \qquad (23)$$

The boundary conditions are

$$r = R_i, \quad T_{s_i} = T_{s,win_i} \tag{24}$$

$$r = R_i + \delta_i, \quad T_{s_i} = T_{s,wout_i} \tag{25}$$

where δ_i means the thickness of the cylindrical layer around each pore where the temperature drops from T_{s,win_i} to $T_{s,wout_i}$. $T_{s,wout_i}$ is a constant temperature outside the solid cylindrical layer. By this we assume also negligible variations of heat flux at the external surface $r = R_i + \delta_i$. This acknowledges the lower scale exact problem where $Bi \lesssim 0.1$. For conjugate problems, the boundary condition between the solid phase and fluid phase is equal temperature and equal heat flux at the interface. So

$$\overline{T}_{f_i}|_{r=R_i} = T_{s_i}|_{r=R_i} = T_{s,win_i} \tag{26}$$

and

$$\begin{aligned} \overrightarrow{q}_{s_i}|_{r=R_i} &= -k_s \frac{\partial T_{s_i}}{\partial r}|_{r=R_i} \\ &= \overrightarrow{q}_{f_i}|_{r=R_i} = -k_f \frac{\partial \overline{T}_{f_i}}{\partial r}|_{r=R_i} \end{aligned} \tag{27}$$

Pore Level Turbulent Regime Equations

In a cylindrical system, for fully developed heat transfer with constant fluid properties and negligible viscous dissipation, the lower scale energy equation for heat transfer in each pore can be written as

$$\begin{aligned} &\overline{u}_i(r)\ r\frac{\partial \overline{T}_{f_i}(x,r)}{\partial x} \\ &= \frac{\partial}{\partial r}\left[r\left(\frac{k_f}{c_{pf}\ \varrho_f} + \frac{\nu_T}{\mathrm{Pr}_T}\right)\frac{\partial \overline{T}_{f_i}(x,r)}{\partial r}\right] \end{aligned} \tag{28}$$

A solution procedure for Eq. (28) can be found in Mills (1995)

The symmetry boundary condition is applied to Eq. (28) at the center line of the straight pore

$$r = 0, \quad \frac{\partial \overline{T}_{f_i}(z,r)}{\partial r} = 0. \tag{29}$$

The conjugate boundary condition is

$$\overline{T}_{f_i}|_{r=R_i} = T_{s_i}|_{r=R_i} = T_{s,win_i}(z) \tag{30}$$

and

$$\overrightarrow{q}_{s_i}|_{r=R_i} = \overrightarrow{q}_{f_i}|_{r=R_i} \tag{31}$$

This means that the solid phase and fluid phase have the same temperature and the same heat flux at the interface.

Relationship between lower and upper scale temperatures

Figure 2 shows the relationship between the lower scale variables and upper scale variables. The notation used in the figure are explained in the following text.

The average over the fluid phase within the REV is

$$\begin{aligned} \langle \overline{T}_f \rangle_f &= \frac{1}{\Delta\Omega}\int_{\Delta\Omega_f} \overline{T}_f d\omega \\ &= \langle m \rangle \left\{\overline{T}\right\}_f = \langle m \rangle \widetilde{\overline{T}}, \end{aligned} \tag{32}$$

and the average over the solid volume within the REV is

$$\begin{aligned} \langle T_s \rangle_s &= \frac{1}{\Delta\Omega}\int_{\Delta\Omega_s} T_s d\omega \\ &= (1 - \langle m \rangle)\left\{T_s\right\}_s = (1 - \langle m \rangle)\widetilde{T}_s \end{aligned} \tag{33}$$

where $\left\{\overline{T}_f\right\}_f = \frac{1}{\Delta\Omega_f}\int_{\Delta\Omega_f} \overline{T}_f d\omega$ and $\left\{T_s\right\}_s = \frac{1}{\Delta\Omega_s}\int_{\Delta\Omega_s} T_s d\omega$ are another definition for intrinsic variable. The fluid temperature can be represented by double, triple and quadruple decompositions suited for VAT needs (Scherban et al., 1986; Primak et al., 1986).

$$\begin{aligned} T_f &= \overline{T}_f + T' \\ &= \widetilde{\overline{T}}_f(x,r) + T'_{fr}(x,r) + T'_f(x,r,t) \\ &= \left\{\overline{T}_{fk}(x,r) + \widetilde{\overline{T}}_{fr}(x,r) + T'_{fr}(x,r)\right\} \\ &\quad + T'_{fk}(x,r,t) \end{aligned} \tag{34}$$

For turbulent flow

$$T'_{fk}(x,r,t) = T'_i(x,r,t) \tag{35}$$

Since pores are nonintersecting and it is assumed that in each pore i, the temperature fields T_{f_i} and T_{s_i} are independent of other fields, the problem can be simplified to

$$\widetilde{T}_{fr}(x,r) = 0 \tag{36}$$

and in each pore

$$T_{fi} = \left\{\overline{T}_{fk}(x) + \widehat{\overline{T}}_{fi}(x,r)\right\} + T'_i(x,r,t) \tag{37}$$

$$T'_{fr}(x,r) = \widehat{\overline{T}}_{fi}(x,r) \tag{38}$$

For each individual pore, the properties can be written as

$$\overline{T}_{fk}(x) = \sum_{i=1}^{N} \overline{\overline{T}}_{fk_i}(x)\, S_i/S_{cr} \tag{39}$$

where the mean cross section temperature is found from

$$\begin{aligned} \overline{\overline{T}}_{fk_i}(x) &= \frac{1}{\Delta\Omega_i}\int_{\Delta\Omega_i} \overline{T}_{f_i}(x,r)\, d\omega \\ &= \frac{1}{S_i}\int_{S_i} \overline{T}_{f_i} ds \\ &= \frac{1}{\pi R_i^2}\int 2\pi r \overline{T}_{f_i} dr \end{aligned} \tag{40}$$

with

$$S_i = \pi R_i^2. \tag{41}$$

$$S_{cr} = \sum_{i=1}^{N} S_i \tag{42}$$

An exact profile of the temperature in each pore is given by

$$\begin{aligned} \overline{T}_{f_i}(x,r) &= \widetilde{\overline{T}}_f(x) + \widehat{\overline{T}}_{f_i}(x,r) \\ &= \overline{T}_{fk}(x) + \widehat{\overline{T}}_{f_i}(x,r). \end{aligned} \tag{43}$$

The temperature field will be represented by these components in each of the elementary elements of the REV. If bipore morphology (Fig. 2) is taken a into consideration, the turbulent regime temperature in each pore of a bipore morphology with radius r_i is given by

$$\overline{T}_{f_i} = \widetilde{\overline{T}}_f + \widehat{\overline{T}}_{f_i}, \qquad i = 1,2 \tag{44}$$

When the average in the REV is defined over the averaging volume $\Delta\Omega_f = \Delta\Omega_{f1} + \Delta\Omega_{f2}$, the averaged temperature is given by

$$\begin{aligned} \widetilde{\overline{T}}_f &= \frac{1}{\Delta\Omega_f}\int_{\Delta\Omega_{f1}} \overline{T}_{f_1} d\omega \\ &\quad + \frac{1}{\Delta\Omega_f}\int_{\Delta\Omega_{f2}} \overline{T}_{f_2} d\omega \\ &= \frac{1}{R_1^2 + R_2^2}\left(R_1^2 \overline{\overline{T}}_{fk_1} + R_2^2 \overline{\overline{T}}_{fk_2}\right) \end{aligned} \tag{45}$$

where

$$\begin{aligned} \overline{\overline{T}}_{fk_1} &= \frac{1}{\pi R_1^2}\int_0^{R_1} \overline{T}_{f_1} \cdot \pi \cdot 2r dr \\ &= \frac{2\pi}{\pi R_1^2}\int_0^{R_1} \left(\widetilde{\overline{T}}_f + \widehat{\overline{T}}_{f_1}\right) r dr \\ &= \widetilde{\overline{T}}_f + \frac{2}{R_1^2}\int_0^{R_1} \widehat{\overline{T}}_{f_1} r dr \\ &= \widetilde{\overline{T}}_f + b_1 \end{aligned} \tag{46}$$

Following the same procedure, we can write

$$\overline{\overline{T}}_{fk_2} = \widetilde{\overline{T}}_f + \frac{2}{R_2^2}\int_0^{R_1} \widehat{\overline{T}}_{f_2} r dr = \widetilde{\overline{T}}_f + b_2. \tag{47}$$

The average of the fluctuation function within the REV is

$$\begin{aligned} \left\langle \widehat{\overline{T}} \right\rangle_f &= \frac{\langle m \rangle}{\Delta\Omega_f}\left(\int_{\Delta\Omega_{f1}} \widehat{\overline{T}}_{f_1} d\omega + \int_{\Delta\Omega_{f2}} \widehat{\overline{T}}_{f_2} d\omega\right) \\ &= \frac{\langle m \rangle}{R_1^2 + R_2^2}\left(R_1^2 b_1 + R_2^2 b_2\right) \end{aligned} \tag{48}$$

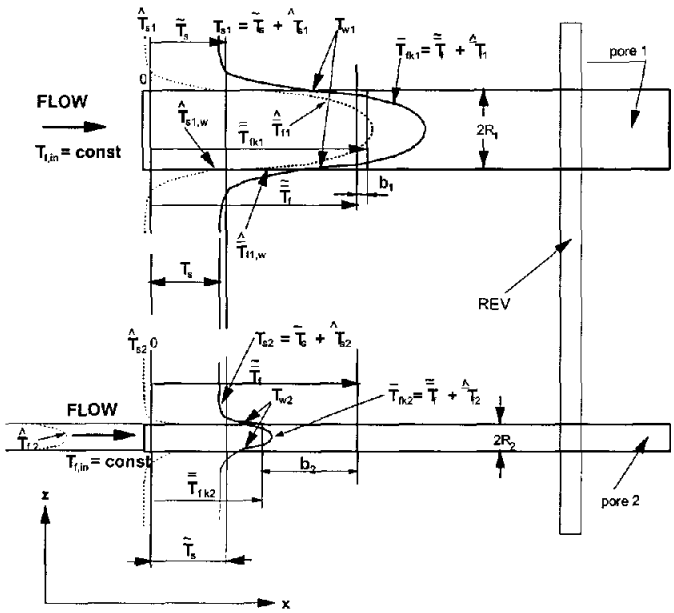

Figure 2: Temperature bulk and local evaluation in a single pore parallel capillary morphology model of porous medium

It is obvious that

$$\left\langle \widehat{\overline{T}} \right\rangle_f = 0 \tag{49}$$

So we can easily derive

$$R_1^2 b_1 + R_2^2 b_2 = 0 \tag{50}$$

Results and Discussion

A porous media network approach is used where the lower scale transport equations are solved first because the lower scale equations have much simpler forms and some of them have theoretical solutions. When the lower scale equations are solved, their solutions can be used to solve the upper scale equation using the relationships between lower scale variables and upper scale variables. The upper scale equation solutions can be used to find the closure models for the upper scale network VAT equations. Finally this approach can be expanded to a general network study.

At this stage a two pore structure is used as a morphology model for a preliminary numerical study. The parameters chosen here are the following: $L = 300mm$, $R_1 = 3mm$, $R_2 = 0.5mm$. Water with inlet temperature $T_{fin} = 80\ ^oC$ is assumed to be flowing through pores with an outside wall temperature $T_{s,wout} = 20^oC$. The solid phase is Aluminum.

To prove that upper scale VAT equations are appropriate for the study of transport phenomena of capillary network, the lower scale equations turbulent momentum and heat transport equations (10), (28) and (23) are solved for $\overline{u}_i(r)$, $\overline{T}_{f_i}(x,r)$ and $T_{s_i}(x,r)$. These lower scale results will be used to show the balance of upper scale VAT equations and to find closures for those unknown terms. Figure 3 shows the lower scale fluid phase and solid phase temperature profile development in pore 1.

When the lower scale equations are solved, the upper scale volume averaged variables $\langle \overline{u} \rangle$, $\langle \overline{T}_f \rangle$ and $\langle T_s \rangle$ can be calculated using Eqs. (8), (45) and (33). Figure 4 shows the volume averaged upper scale one dimensional fluid phase temperature and solid phase temperature profiles along the flow direction. After calculating these averaged variables, the fluctuation variables $\widehat{\overline{T}}_f$ and $\widehat{\overline{u}}$ used in upper scale equations could easily be calculated.

Next, we estimate the unknown integration and differential terms in the upper scale VAT equations so that the balance of both sides of the VAT equations could be checked. For instance, Table 1 lists all the terms in VAT fluid phase heat transfer laminar regime equation.

Since all the variables in these terms listed in Table 1 have been calculated, these terms can easily be estimated. Figure 5 shows the magnitude of each term along the flow direction.

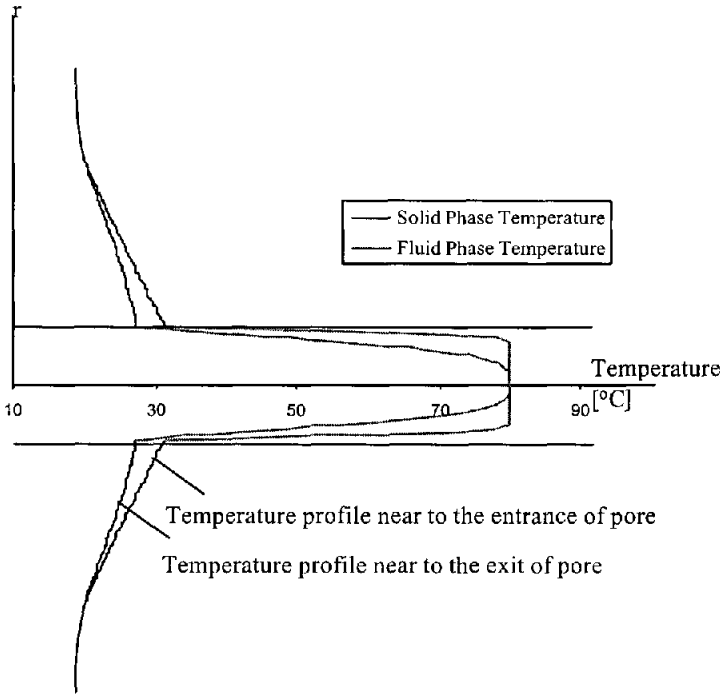

Figure 3: Lower scale temperature distribution of a pore

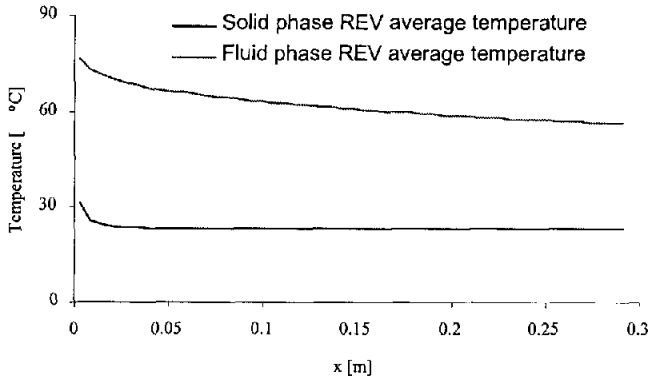

Figure 4: Upper scale voulume averaged temperature distribution

Term	Expression	Physical mean
left	$c_{pf}\varrho_f \langle m\rangle \widetilde{\widetilde{U}}\frac{\partial \widetilde{T}_f}{\partial x}$	convection
right1	$\frac{\partial}{\partial x}\left[\langle m\rangle k_f \frac{\partial \widetilde{T}_f}{\partial x}\right]$	conduction
right2	$c_{pf}\varrho_f \frac{\partial}{\partial x}\left[\langle m\rangle \left\{-\widehat{T}_f\ \widehat{u}\right\}_f\right]$	porous fluctuation
right3	$\frac{\partial}{\partial x}\left[\frac{k_f}{\Delta\Omega}\int_{\partial S_w}\widehat{T}_f\ \vec{ds}\right]$	porous fluctuation
right4	$\frac{1}{\Delta\Omega}\int_{\partial S_w} k_f \frac{\partial T_f}{\partial x_i}\cdot \vec{ds}$	heat exchange

Table 1: Terms in VAT fluid phase heat transfer equation

The "+" and "-" signs in the legend mean the value of the corresponding term be positive and negative respectively. It is clear that the dominant terms are two that are normally seen in this type of modeling and one VAT based term that involves the fluctuations. The VAT type term rapidly decreases with tube length. This will be an important term if capillary junctions are addressed and they are short.

Figure 6 shows the perfect balance of VAT fluid phase heat transfer equation. It demonstrates that approaches are valid for studies of transport phenomena in capillary network.

As long as the closures for the upper scale VAT equations are available, the ensemble properties for flow and heat transfer in a capillary network are calculated directly from the upper scale VAT equations.

Summary and Conclusions

A two scale heat transport conjugate problem in capillary porous medium is addressed using a rigorous application of volume averaging theory. The use of two scale equations for a network problem is an effective contribution to model development for flow in porous medium. The two scale approach is carried out in two steps. The first step is to derive equations for lower scale transport in each straight pore. The second step is to use VAT equations for upper

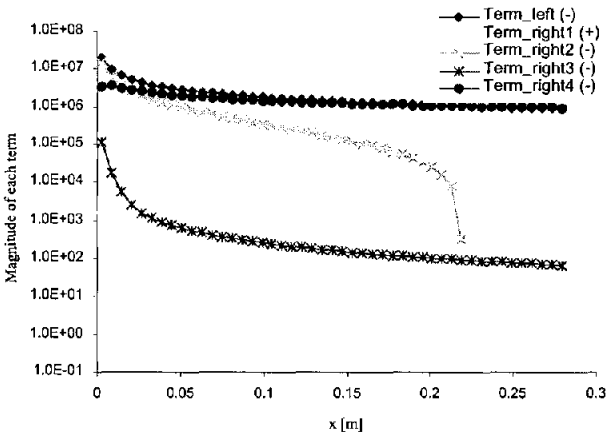

Figure 5: Estimation of terms in upper scale fluid phase heat transfer VAT equation

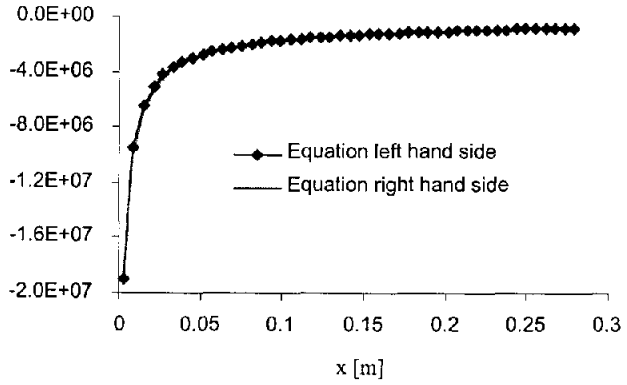

Figure 6: Balance of the upper scale heat transfer VAT equation in fluid phase

scale transport in porous media and properly close them. The lower scale results are used to derive the closures for the upper scale VAT equations.

The problem stated and treated in this paper is part of the problem of capillary network porous medium simulation. Transport in straight pore capillary morphology could be used as a model of membrane transport. The attractive feature of this morphology is that all parameters and characteristics can be evaluated precisely for both scales. This gives one the ability to develop a hierarchical analysis of momentum and heat transport that depends on morphological characteristics.

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